Appraisal Essentials

Residential Site Valuation and Cost Approach

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section one

LAND

LESSON 1: Land/Site Geological and Geographical Characteristics

LESSON 2: The Neighborhood

LEARNING OBJECTIVES

By the end of this section, participants will be able to

- comprehend basic real property concepts;
- understand real property characteristics;
- identify the significance of land in real estate;
- demonstrate familiarity with how various disciplines view the concept of land;
- define an estate;
- identify the characteristics of land; and
- define a neighborhood.

KEY TERMS

accessibility
air rights
appurtenance
assemblage
estate
excess land
external influences
external site improvements
internal site improvements
land
land analysis
land description
linkage
littoral rights
locational influence
mineral rights
neighborhood
plottage
raw land
real estate
real property
riparian rights
site
site improvements
subsurface rights
surface rights
surplus land
topographic map
topography
vacant land
water rights

SUGGESTED READING

Basic Appraisal Principles, Mark A. Munizzo and Lisa Virruso Musial, Dearborn Real Estate Education (2007)

Basic Appraisal Procedures, Mark A. Munizzo and Lisa Virruso Musial, Dearborn Real Estate Education (2007)

LESSON 1: Land/Site Geological and Geographical Characteristics

LAND

Under all is land! It is essential to life. In fact, land provides the foundation for all of the social, political, and economic activity of people, and is the basis for the natural wealth of a nation. Further, land is a physical commodity and the resource from which everything is derived. Land is so important a concept in such disciplines as geography, sociology, law, and economics, that it transcends any one particular definition.

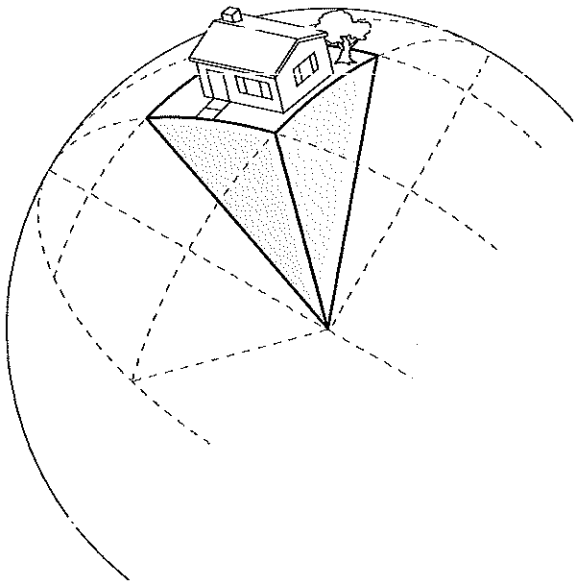
A more qualified definition of land warrants an investigation on how the concept of land is broached in these varied disciplines. In geography, land is seen as having physical elements of which people make use. In sociology, land is seen as a shared resource, or a commodity to be owned, traded, or individually used. In law, the issues of ownership and use are considered. In economics, land is seen as one of the four agents of production (land, labor, capital, and coordination).

Although each of these disciplines may have a different focus concerning land, there is common agreement about the attributes of land.

- Land has use.
- Land is durable.
- Land is physical.
- Land is immobile.
- Land has a finite supply.
- Each parcel of land is unique in its location.
- Each parcel of land is unique in its composition.

Land includes the ground and the soils as well as everything attached to the earth, whether by nature (as in the trees) or subsurface minerals or by the hand of man (as in improvements such as buildings or fences). Land therefore is like an inverted pyramid with an apex to the center of the earth, continuing upward to the earth's surface in the shape of the parcel and extending outward from the surface to the sky into infinity; however, it is worth noting that the U.S. Congress has declared that there is a "public

FIGURE 1.1
Land Definition



right to freedom of transit" for commerce in the airspace over the United States. Therefore the federal government has sovereign right over the nation's airspace. (See Figure 1.1.)

Real Estate and Real Property

The distinction between real estate and real property is important to appraisers. In some state laws and certain courts, the terms are viewed as synonymous. Likewise, in some disciplines and amongst some real estate professionals the terms are interchangeable. In the appraisal profession the terms are distinct, and it should be made clear to keep these terms separate.

Real estate is the physical land and the improvements affixed to the land. Such improvements are sometimes called **appurtenances** and they refer to the items affixed to the land that have become a physical part of the land as real estate.

Real property refers to the inherent rights of ownership in real estate, including all benefits and interests associated with ownership. The rights of ownership are also commonly referred to as an **estate.** This is a legal consideration, but affects the value of the property nonetheless.

Surface, Subsurface (Mineral), Air, and Water Rights

Property ownership includes the right to use the surface of the earth, referred to as **surface rights.** The right to use the *subsurface* of the earth, referred to as **subsurface rights,** includes any natural resources lying beneath the earth's surface that may or may not be included. For example, any subsurface rights to the minerals beneath the earth's surface are known

as **mineral rights.** It is possible for an owner to transfer the surface rights, while retaining the subsurface rights. Such rights can be leased, sold, or deeded for a certain period of time or in *perpetuity*, which means to continue forever. These rights can also be acquired as an inheritance by will.

The right to use the space above the surface of the earth is referred to as **air rights.** Prior to air travel, ownership rights automatically extended toward space; however, in air travel times, air rights have become limited. The U.S. Congress has placed limitations on air rights in order to accommodate aircrafts, spacecrafts, and other digital devices that use open airwaves.

Water rights are common-law rights as well as statutory rights, depending upon the state, held by owners of land near bodies of water, such as lakes, rivers, or oceans. The rights associated with ownership of lands bordering such bodies of water, are known as *riparian* and *littoral* rights. **Riparian** rights may include construction of improvements along or over a waterway, such as piers and boat slips, or the use of waterway for fishing or other recreational purposes. **Littoral rights** are for the enjoyment of the shoreline and safeguard against the interference that my change the shoreline.

Because water is such an important commodity to life, water rights are ordinarily restricted and are especially important to areas where water is scarce and needed to accommodate the needs of the general public. Water rights can be very complex and an appraiser must be competent in this area in order to complete an assignment involving such rights.

LAND VERSUS SITE

Appraisers are sometimes engaged to value the land only, or the real estate, which would include the land and all improvements upon the land. Land with no improvements is known as **raw land** or sometimes called *unimproved land.* Raw land can be land that is simply undeveloped or put to an agricultural use. When land has had some improvements that would support a specific use, then it is said to be a **site.** The term *site* is specific and it means that there are improvements that would allow for further appurtenances to be developed. As an example, an industrial site is vacant land that would allow for truck access, loading docks, and possesses the necessary utilities to support an industrial building.

Vacant land can either be vacant raw land or a vacant site, and it is the responsibility of the appraiser to clarify what type of vacant land is being analyzed.

Land analysis consists of factual information that pertains to the neighborhood and market characteristics that create, or impact, the utility and

Plottage

Plottage is the incremental value increase that is commensurate with the additional utility created by combining two or more parcels under a single ownership. The act or process of combining two or more parcels under a single ownership to create a larger tract with greater utility is called **assemblage**.

Property Description and Location

No matter what the property consists of, land or site, geological or geographical considerations, zoning, environmental, and social implications all have a huge influence on how and for what the property will be used. In short, the real estate world uses the simple term "location." The concept of location virtually means taking all of these issues—and more—into account, and noting their effects on the value of the property.

Perhaps the most important location category is the concept of "neighborhood." This simple concept dictates the use, linkage, amenities, and structural boundaries of an area. The boundaries are typically drawn based on geographical, political, or complementary uses of the properties within the neighborhood.

LESSON 1 REVIEW QUESTIONS

1. An improved parcel of land ready for a particular use is known as a _____.
2. Land on which *no* improvements have been made is known as _____ _____ or _____.
3. The surface contour of land is known as _____.
4. Increase in value as a result of combining parcels under a single ownership is known as _____.
5. Land that is above and beyond the land needed for the existing improvements, and does not support expansion or a separate use, is known as _____ land.
6. _____ relationship is known as linkages.
7. A property that is located on a corner may suffer or benefit from its _____.
8. Land that is above and beyond what is needed for the improvements or the highest and best use, and can be separated or used to support future expansion, is known as _____ land.
9. To combine two or more parcels is known as _____.
10. The influences that are _____ of the property lines are known as external influences.

LESSON 1 REVIEW ANSWERS

1. site
2. raw land; unimproved land
3. topography
4. plottage
5. surplus
6. Time-distance
7. locational influence
8. excess
9. assemblage
10. outside

LESSON 2: The Neighborhood

THE NEIGHBORHOOD

"It's a beautiful day in the neighborhood," but what is it that defines, determines, or delineates the composition and boundaries of a neighborhood? A **neighborhood** is defined as a geographic area delineated by geographic or political boundaries that is characterized by having complementary land uses. So what does this actually mean to the practicing appraisal professional? If the appraiser is required to define the neighborhood, then what implications does this definition have on the appraisal process, scope of work, and so on? The neighborhood is critical to the market information that is relevant to the subject.

How do you determine the neighborhood? Well, the information and data that are uncovered during the market analysis process are the supportive proof of what the market indicates are the forces that have an effect on the property based on location. In other words, market analysis and the critical thought process concerning the data uncovered reflect what the market sees are the location factors of the area.

Natural Boundaries vs. Economic Boundaries

Neighborhoods are delineated by physical boundaries such as roads, rivers, railroad lines, and the like. The name of a neighborhood might not be enough to describe the area accurately. There may be an economic difference within the same neighborhood by name, and yet both districts would carry the same name in a report. It is important to remember that a single subdivision may or may not make up neighborhood. In fact, a neighborhood could be comprised of several subdivisions and, conversely, only part of a subdivision may be a neighborhood. In such a "split-neighborhood" market, the appraiser must maintain caution when choosing information, particularly sales comparables, from such an area. A more detailed explanation of the neighborhood boundaries beyond the name of the district and based on the economic trends of both areas within the district is required for clarity.

The important issue that an appraisal professional must remember about the "neighborhood" is that the area should speak to the value or market implications. This can only be done with market analysis and supportive data extracted from the market using value and trend indicators.

The Protected Classes

Race, religion, familial status, sexual preference, and all other protected classes are not appropriate neighborhood descriptions. While it may be true that we as human beings tend to socialize or habitate with others of our own views, beliefs, and so on, such issues of identification, particularly in the neighborhood analysis, are irrelevant and can easily be construed as discriminatory in nature. An appraiser must be very careful to identify only the relevant issues of a neighborhood composition that drive value and marketability.

Determining the neighborhood is usually fairly obvious, but support for what the appraiser deems is a market area is critical to this decision. Descriptive terms such as "urban," "rural," "suburban," "town," "village," "city," and "metropolis" or "metropolitan district" should be accurate and clearly defined within the reporting process of the assignment.

LESSON 2 REVIEW QUESTIONS

1. A/an _____ is defined as a geographic area delineated by geographic or political boundaries that is characterized by having complementary land uses.

2. A neighborhood can either be described by either _____ or _____ boundaries.

3. Using the protected classes to describe a neighborhood is irrelevant and can easily be construed as _____ in nature.

4. Market analysis and the critical thought process concerning the data uncovered will reflect what the market sees are the _____ factors of the area.

5. There may be an economic difference within the same neighborhood having two separate districts. This is known as a/an _____ market.

LESSON 2 REVIEW ANSWERS

1. neighborhood

2. economic; physical

3. discriminatory

4. location

5. split-neighborhood

SECTION 1 REVIEW QUESTIONS

1. Real estate is the physical land and
   a. improvements.
   b. real estate.
   c. water.
   d. real property.

2. Land that has some improvements that would support a specific use is known as
   a. a site.
   b. a vacant parcel.
   c. a lot.
   d. a subdivision.

3. The surface contour of land is known as
   a. density.
   b. topography.
   c. plottage.
   d. assemblage.

4. The incremental value increase that is a result of combining two or more parcels of land is
   a. assemblage.
   b. plottage.
   c. density.
   d. a subdivision.

5. The act or process of combining two or more parcels under a single ownership is
   a. assemblage.
   b. plottage.
   c. density.
   d. a subdivision.

6. Land that has no improvements is known as
   a. a site.
   b. raw land.
   c. a lot.
   d. a subdivision.

7. A neighborhood is defined as
   a. having geographic boundaries.
   b. having political boundaries.
   c. complementary land uses.
   d. All of the above

8. The incremental increase in value as a result of assembling properties together is known as
   a. assemblage.
   b. plottage.
   c. percolation.
   d. None of the above

9. Race and religion are considered
   a. relevant neighborhood characteristics.
   b. good market indicators.
   c. irrelevant to neighborhood description.
   d. indicators of value.

section two

II

LAND OR SITE VALUATION TECHNIQUES

LESSON 1: Sales Comparison Method

LESSON 2: Allocation Method

LESSON 3: Extraction or Abstraction Method

LESSON 4: Subdivision Development Method

LESSON 5: Ground Rent Capitalization Method

LESSON 6: Land Residual Method

LEARNING OBJECTIVES

By the end of this section, participants will be able to

- identify the six land or site valuation techniques;
- value land or site using all six valuation techniques;
- understand the sequence of adjustments; and
- distinguish between percentage adjustments and lump sum adjustments.

KEY TERMS

absorption rate
abstraction method
allocation method
discounting
element of comparison
extraction method
ground lease
ground rent capitalization method
land residual method
sales comparison method
subdivision development method
unit of comparison
paired sales analysis

SUGGESTED READING

Basic Appraisal Principles, Mark A. Munizzo and Lisa Virruso Musial, Dearborn Real Estate Education (2007)

Basic Appraisal Procedures, Mark A. Munizzo and Lisa Virruso Musial, Dearborn Real Estate Education (2007)

LESSON 1: Sales Comparison Method

The last step before reconciling to an indicated value conclusion under the cost approach is to add the value of the land to the depreciated cost new of the property. Since land does not depreciate, the land value is added after depreciation has been removed from Reproduction or Replacement Cost New of the property. In the cost approach one may either add the cost of land plus the cost of the site improvements, or simply add the cost of the site. Either way is acceptable provided that the reader clearly understands which method was performed, and that all of the costs are accounted for without double counting the site improvements.

Land or site valuation techniques are used even if the cost approach is not used. In other words, the appraisal assignment may be just to value vacant land or a site.

With the **sales comparison method**, sales of vacant sites are obtained from the market and compared to the subject. A final estimate of value for the subject site as though vacant is then arrived at after any necessary adjustments are made for differences. This method is considered to be the most reliable and most preferred method to value land or sites. The sales comparison approach is most useful when there are an ample number of similar sales to the subject property (an active market) that have recently sold as arm's-length transactions (transactions between two or more unrelated parties acting independently of each other).

When there is a lack of comparable sales data, other methods are used. These methods, discussed in the following lessons, can also be used to support a value conclusion by the sales comparison method.

The process in the sales comparison approach is to itemize the subject and comparable sales into *elements of comparison* whereby adjustments are made to the sales price of the comparable based on the differences between that element.

ELEMENTS OF COMPARISON

Elements of comparison are categorized characteristics of properties that explain the variances in property values. When appraising a parcel of land or site, considered elements of comparison would include:

1. Property rights conveyed
2. Financing
3. Conditions of sale
4. Market conditions (time)
5. Location
6. Physical characteristics
7. Zoning and land-use restrictions
8. Availability, capacity, and proximity of utilities (amount of site development)

Property Rights Conveyed

Comparable sales must be adjusted for differences in property rights conveyed. For example, there is a difference if the owner of a comparable sale has a leased fee estate (the property is encumbered by long-term leases) and the owner of the subject has a fee simple estate. These differences can have a significant impact on property value. Additionally, there may be nonrealty interest items, such as personal property, that were included in the sale of a comparable sale.

Financing

Implicit in the definition of market value is that payments to the seller are made in terms of cash or cash equivalency, and that the property is unaffected by special or creative financing or sales concessions. If a comparable sale involved special sales or financing concessions, you must make adjustments—whether positive or negative. Seller financing with terms and conditions different from what is offered in the financing market, special loan types, and usual terms of a loan are all examples of special sales, or financing concessions.

Conditions of Sale

Implicit in the definition of market value is that buyer and seller are typically motivated and that both parties are well-informed and acting in what they consider their best interests. For example, if the appraiser interprets a comparable sale transaction as a sale that occurred under duress, then appropriate adjustments need to be made. In addition, if it is determined that the comparable sale was *not* an arm's-length transaction, adjustments would need to be made. Generally speaking, if the market supplies an abundance of comparable data, then these comparable sales should be omitted.

Market Conditions (Time)

Because the principle of *change* dictates that nothing remains static and that everything changes over time, the date that a sale occurred is extremely important. The greater the difference between the effective date of an appraisal and the sale date of the comparable sale, the greater the potential for significant impacts on value. However, even if a short time has transpired since the comparable sale date and the effective date, a major change on the market economic base can significantly impact values in that market.

For example, even a recent comparable sale, two months prior to the effective date, may need to be adjusted for market conditions. An adjustment may be needed because it was just announced that a large manufacturing plant in the immediate area will be closing. Property values can be significantly impacted due to large layoffs and the subsequent market reaction to the forces that affect value. Likewise, property values can be significantly enhanced if the arrival of a new airport or large business is announced, thus creating employment opportunities.

Location

Under ideal circumstances, the subject and all comparable sales would be located within the same neighborhood. However, this is not always possible; therefore any differences in location, even within the same neighborhood, must be accounted for.

Physical Characteristics

Physical characteristics of land include size, shape, and terrain. The appraiser must assess the site dimensions and determine if the site is adequate for specific uses, and also determine value implications of the site's size and shape. The appraiser must also consider how the size and shape will affect the probable use of the parcel. This is the basis of the *physically possible* test under the highest and best use analysis. When comparing the size and shape of the subject to the size and shape of the comparable sales, it is important to remember that the size and shape of a parcel of land is usually closely associated with the optimal use of that parcel. Other physical characteristics include topography, soil conditions, drainage, view, and frontage.

Zoning and Land-Use Restrictions

Police power is one of the four public restrictions that supersede the rights of an individual owner. Police power allows for zoning ordinances (land-use restrictions), building codes, health codes and regulations, environmental regulations, subdivision development regulations, and historical restrictions. In addition to public restrictions, private restrictions (deed restrictions or restrictive convenants) may also exist and would need to be adjusted for.

Availability, Capacity, and Proximity of Utilities

Land on which no improvements have been made is known as *raw land*, or sometimes *unimproved land*. Raw land can be land that is simply undeveloped or put to an agricultural use. When land has had some improvements that would support a specific use, then it is said to be a *site*. The term *site* is specific and it means that there are improvements that would allow for further appurtenances to be developed. As an example, an industrial site is vacant land that would allow for truck access and loading docks, and possesses the necessary utilities to support an industrial building.

Site improvements such as utilities, curbs, and roads are elements of value and therefore contribute to the value of the land. Adjustments for differences between the subject and all comparables then need to be made accordingly.

FIGURE 2.1
Unit of Comparison Formula

Unit of comparison = Sales price ÷ Element of comparison

ADJUSTMENT PROCESS

The adjustments are first extracted from the market's perceived response to a particular element using a paired sales analysis (a methodized process to extract adjustments), and then the adjustments are applied to the comparable in order to conclude an indication of value for the subject.

When comparing the subject to the comparable sales it is important to compare the same *unit of comparison* or *unit of measure.* A **unit of comparison** is a physical or economic component that reflects the relationship between sales price and various elements of comparison. (See Figure 2.1.) A unit of comparison creates a more standardized method of comparison. Commonly used units of comparison for vacant land are price per square foot, price per acre, and price per front foot.

When applying *percentage adjustments,* you must make the adjustments in a specific order, creating a new base after each element of comparison adjustment. The order is as follows:

1. Property rights conveyed
2. Financing
3. Conditions of sale
4. Market conditions (time)

Generally, adjustments to the first four elements of comparison are adjusted with percentage adjustments, while the remaining elements of comparison are adjusted using lump-sum dollar amounts. With **lump sum dollar adjustments,** the adjustments within each sale are "netted out" and the net result is either added to or subtracted from the sales price to reach an indicated value for each comparable. A **net adjustment** is the sum of all positive and negative adjustments. A shortcoming of net adjustments is that a series of net adjustments can potentially zero each other out. Because of this, *gross adjustments* must be considered. A **gross adjustment** is the result of adding the adjustments for a comparable and ignoring the signs. In other words, it is the total dollar amount of adjustments ignoring whether the adjustment is negative or positive. Gross adjustments indicate whether a comparable sale is reliable. The comparable with the least amount of gross adjustment is usually the most reliable indicator of subject value. Both net and gross adjustments can be expressed as a percentage of the sale price of the comparable sale.

TABLE 2.1
Sales Comparison Table

	Subject	Sale 1	Adjust	Sale 2	Adjust	Sale 3	Adjust	Sale 4	Adjust
Sales Price	N/A	$230,000		$245,000		$235,000		$260,000	
Property Rights	Fee Simple	Fee Simple		Fee Simple		Fee Simple		Fee Simple	
Financing	None	None		None		None		Special Financing	-10% (-$26,000)
Conditions of Sale	None	Equal		Equal		Equal		Equal	
Market Conditions (time)	Current	5 weeks prior		4 weeks prior		5 weeks prior		1 week prior	
Location	Good	Average	+$10,000	Good		Average	+$10,000	Average	+$10,000*
Physical	50' × 100'	Equal		Equal		Equal		Equal	
Zoning	R-1	R-1		R-1		R-1		R-1	
Utilities	Available	Available		Available		Available		Available	
Net Adjust (%)			+$10,000 (4.3%)		$0 (0%)		+$10,000 (4.3%)		-$16,000 (-6.2%)
Gross Adjust (%)			$10,000 (4.3%)		(0%)		$10,000 (4.3%)		$36,000 (13.8%)
Indicated Value			**$240,000**		**$245,000**		**$245,000**		**$244,000**

*Because the financing percentage adjustment must be calculated *first*, the $10,000 location adjustment is added to the new base sales price [($260,000 – $26,000) + $10,000] resulting in a concluded value of $244,000. If the minus 10 percent financing adjustment was calculated *last*, it would be -$27,000 [($260,000 + $10,000) × 10%] , instead of -$26,000.

To remember whether the adjustment to the comparable is a negative or positive adjustment, use the acronyms CIA and CBS: Comparable Inferior Add (CIA), Comparable Better Subtract (CBS).

Whether using lump-sum dollar amount or percentage adjustments, you adjust the comparable to the subject. Therefore, if the comparable is superior to the subject the adjustment must be negative to make the comparable similar to the subject. Likewise, if the comparable is inferior to the subject a positive adjustment must be made to the comparable to make the comparable similar to the subject.

An example of a percentage adjustment is when, after careful consideration of pertinent elements of comparison, the appraiser needs to adjust one of the comparable sales downward by 10 percent for favorable financing. If it is also determined that a positive $10,000 adjustment needs to be made for location because several of the comparables have a location that is considered inferior to the subject, then this would be a lump-sum adjustment.

RECONCILIATION

After the indications of value from each comparable are completed, the appraiser must *reconcile* which of the sales are most similar (*comparable*) to the subject. After this weighting process is completed, the appraiser concludes a value indication based on the value range before and after the adjustment process. Sales that have more adjustments are less similar to the subject, and should be given less weight in the reconciliation process. The sales with the least adjustments are probably more indicative of what the subject's value is likely to be, and should therefore be give greater weight in the indicated value conclusion under the sales comparison method of land or site valuation.

In Practice

Based on the sales grid in Table 2.1, which comparable would you give the most weight to and why? What is the indicated value of the subject property?

Solution: Sale 2 deserves the most weight because it has the least adjustments. The indicated value of the subject is $245,000.

In Practice

Your assignment is to value a residential lot. In your data collection processes, you find four comparable sales that are most similar to the subject. Upon completing a paired sales analysis, you extracted the following adjustments:

1. A lot with a frontage of 100 feet sells for $5,000 more than lots with frontages of 60 to 80 feet.
2. A lot with a good location is 5 percent more valuable than a lot with an average location.
3. A rectangular shaped lot is $6,000 more valuable than an irregular shaped lot.
4. Favorable financing conditions warrant a 5 percent downward adjustment.
5. Residential lots, no matter what size, that are zoned R-3 are $10,000 more valuable than lots that are zoned R-2.

The following sales grid organizes the information about the subject as well as the four comparable land sales. Based on this information, complete the market data grid provided.

	Subject	Sale 1	Adjust	Sale 2	Adjust	Sale 3	Adjust	Sale 4	Adjust
Sales Price	N/A	$330,000		$340,000		$335,000		$350,000	
Property Rights	Fee Simple	Fee Simple		Fee Simple		Fee Simple		Fee Simple	
Financing	None	None		Special Financing		None		None	
Conditions of Sale	None	Equal		Equal		Equal		Equal	
Market Conditions (time)	Current	2 months prior		1 month prior		3 weeks prior		1 week prior	
Location	Good	Average		Average		Good		Good	
Shape	Rectangular	Rectangular		Irregular		Irregular		Rectangular	
Dimensions	80' × 180'	100' × 175'		80' × 180'		100' × 180'		100' × 180'	
Zoning	R-3	R-3		R-2		R-2		R-3	
Net Adjust (%)									
Gross Adjust (%)									
Indicated Value									

Solution:

	Subject	Sale 1	Adjust	Sale 2	Adjust	Sale 3	Adjust	Sale 4	Adjust
Sales Price	N/A	$330,000		$340,000		$335,000		$350,000	
Property Rights	Fee Simple	Fee Simple		Fee Simple		Fee Simple		Fee Simple	
Financing	None	None		Special Financing	–5% = –$17,000	None		None	
Conditions of Sale	None	Equal		Equal		Equal		Equal	
Market Conditions (time)	Current	2 months prior		1 month prior		3 weeks prior		1 week prior	
Location	Good	Average	+5% +$16,500	Average	+5% = $16,150*	Good		Good	
Shape	Rectangular	Rectangular		Irregular		Irregular	+$6,000	Rectangular	
Dimensions	80' × 180'	100' × 175'	-$5,000	80' × 180'	+$6,000*	100' × 180'	-$5,000	100' × 180'	-$5,000
Zoning	R-3	R-3		R-2		R-2	+$10,000	R-3	
Net Adjust (%)			$11,500 (+3.4%)		$5,150 (+1.5%)		$11,000 (+3.3%)		-$5,000 (–1.4%)
Gross Adjust (%)			$21,500 (6.5%)		$39,150 (11.51%)		$21,000 (6.3%)		$5,000 (1.4%)
Indicated Value			**$341,500**		**$345,150**		**$346,000**		**$345,000**

*Because the financing percentage adjustment must be calculated first, the 5 percent location adjustment is added to the new base sale price ($340,000 – $17,000 or $340,000 × 0.95) resulting in a location adjustment of +$16,150, ($323,000 × 5%). Then add the $6,000 lot shape adjustment to the new base sales price of $339,150, ($323,000 × 1.05).

■ In Practice

Based on the sales grid in the previous In Practice section, which comparable would you give the most weight to, and why? What is the indicated value of the subject property?

Solution: Sale 4 would carry the most weight because it has the least adjustments. The indicated value of the subject is $345,000.

LESSON 1 REVIEW QUESTIONS

1. Which method of land or site valuation is considered to be the most reliable and preferred method?
   a. Allocation method
   b. Extraction method
   c. Sales comparison method
   d. Land residual method

2. Which of the following formulas calculates a unit of comparison?
   a. Sales price ÷ element of comparison
   b. Element of comparison ÷ sales price
   c. Sales price × 50%
   d. None of the above

3. If an element of the comparable is *superior* to the subject then
   a. there is a negative adjustment made to the subject.
   b. there is a positive adjustment made to the subject.
   c. there is a negative adjustment made to the comparable sale.
   d. there is a positive adjustment made to the comparable sale.

4. If an element of the comparable is *inferior* to the subject then
   a. there is a negative adjustment made to the subject.
   b. there is a positive adjustment made to the subject.
   c. there is a negative adjustment made to the comparable sale.
   d. there is a positive adjustment made to the comparable.

5. In which order should the adjustments be made when making percentage adjustments?
   a. Property rights conveyed, location, time, physical characteristics
   b. Property rights conveyed, financing, conditions of sale, market conditions (time)
   c. Lot size, market conditions, topography, soil conditions
   d. Lot size, topography, shape, financing

LESSON 1 REVIEW ANSWERS

1. c
2. a
3. c
4. d
5. b

LESSON 2: Allocation Method

The **allocation method** is based on the principle of balance. The principle of balance states that there is a proportional relationship between the four agents of production; land, labor, capital, and coordination. Because land is one of these agents, there should be a relationship between land value and total property value. When a property is improved, applying the ratio of land to building value of similar properties in the market to the subject-improved property will provide the land value of the subject.

For example, if the typical ratio of land to building is one to five (expressed as L:B = 1:5), and a parcel improved with a structure has a *total* estimated value of $200,000, then the land is one-sixth (there are six total parts) of that total value, or $33,333. This translates into land percentage of 16.67 percent, (1 ÷ 6 or $33,333 ÷ $200,000). Thus part of the total property value is *allocated* to the site and part is allocated to the improvements.

In Practice

The typical ratio of land to building is 1:5, and the total estimated *building* value is $250,000. Using this information, answer the following questions:

1. What is the estimated land value?
2. What is the total property value?
3. What is the land percentage?
4. What is the building percentage?

Solution: Because there are five parts allocated to the building value only, the first step is to determine the value of one part. Remember, there are a total of six parts.

1. $250,000 ÷ 5 = $50,000, because the land value is equal to one part, the estimated land value = $50,000
2. The total property value = $300,000 ($250,000 + $50,000)
3. Land percentage = 0.167 or 16.7% (1 ÷ 6 or $50,000 ÷ $300,000)
4. Building percentage = 0.833 or 83.3% (5 ÷ 6 or $250,000 ÷ $300,000)

The allocation method can be used if there is adequate comparable sales data available. When applying the allocation method, it is assumed that there is a typical land to building ratio for similar property types in a similar location.

A shortcoming of this method is that it can be inaccurate because it uses average (typical) ratios for a particular type of property. The actual configuration (building sizes and appeal) of particular property types with similar values can differ substantially. The allocation method does not take these differences into account. Therefore, if used alone, the allocation method is not a reliable indicator of land or site value; however, it can be used to support a land or site value conclusion using other techniques discussed in this section or when it would be impractical to value a mass amount of properties with land and improvements segregated, such as in the case of property tax assessment.

If there are no sales of vacant land in the subject's neighborhood, then the following steps need to be completed in order to perform the allocation method of land or site valuation:

- Find average *improved* property sales and *vacant* land sales with a similar neighborhood as the subject.
- The typical vacant land sales price is divided by the sales price of improved properties. This results in a land value to total property value ratio.
- The land value to total property ratio is then multiplied by the sales price of improved comparables sales located in the same neighborhood as the subject. This results in a land value range for the subject.

In Practice

The following information was gathered from closed sales of vacant land as well as improved property, located in a *similar* neighborhood as your subject property. The typical sales price of improved comparables located in the same neighborhood as the subject range between $565,000 and $580,000. What is the land value range for the subject?

Vacant land sales:

Sale Number	Sales Price
1	$146,000
2	$141,000
3	$141,500
4	$142,700

Improved property sales:

Sale Number	Sales Price
1 (high)	$585,000
2	$579,000
3 (low)	$568,000
4	$571,500
5	$575,000

Solution: First, determine the typical land sales price by determining the mean of the four sales:

$146,000 + $141,000 + $141,500 + $142,700 = $571,200

$571,200 ÷ 4

= $142,800

To determine the land value range, first divide the typical land sales price (the mean) by the highest and lowest improved property sales.

$142,800 ÷ $585,000 (highest) = 24%

$142,800 ÷ $568,000 (lowest) = 25%

In this example, the land value range is approximately 24 percent to 25 percent of the total property value.

To determine the land value range for the subject, multiply the land value range percentages by the highest and lowest improved property sales in the subject neighborhood.

$580,000 × 0.24 = $139,200

$565,000 × 0.25 = $141,250

Therefore, the land value range for the subject would be from $139,200 to $141,250.

LESSON 2 REVIEW QUESTIONS

1. The allocation method
   a. is not a realizable indicator if used alone.
   b. can be used to support a land or site value conclusion using other techniques.
   c. is sometimes used in mass appraisal.
   d. All of the above

2. If the typical land to building ratio is 1:4, then land is
   a. ¼ of the total building value.
   b. ⅕ of the total property value.
   c. ¼ of the total property value.
   d. ⅕ of the total building value.

3. If the typical land to building ratio is 1:3 then
   a. the building is ¾ of the property value.
   b. land is ¼ of the total property value.
   c. Both a and b
   d. None of the above

4. If the typical ratio of land to building is 1:6 and the total estimated total property value (land plus building) is $600,000, what is the estimated land value?
   a. $66,666
   b. $85,714
   c. $50,000
   d. $120,000

5. Using the information in question 4, what is the building value?
   a. $66,666
   b. $85,714
   c. $514,284
   d. Cannot be determined

6. Using the information in question 4, what is the land percentage?
   a. 16.67%
   b. 14.29%
   c. 85.71%
   d. 83.33%

7. Using the information in question 4, what is the building percentage?
   a. 16.67%
   b. 14.29%
   c. 85.71%
   d. 83.33%

8. The typical ratio of land to building is 1:5, and the total estimated total property value (land plus building) is $500,000. What is the estimated land value?
   a. $125,000
   b. $50,000
   c. $100,000
   d. $83,333

9. Using the information in question 8, what is the estimated building value?
   a. $416,665
   b. $50,000
   c. $100,000
   d. $83,333

Please use the following information to answer question 10.

The information was gathered from closed sales of vacant land as well as improved property, located in a neighborhood *similar* to your subject property. The typical sales price of improved comparables located in the same neighborhood as the subject range between $855,000 and $890,000.

Vacant land sales:

Sale Number	Sales Price
1	$225,000
2	$210,000
3	$227,000

Improved property sales:

Sale Number	Sales Price
1 (high)	$885,000
2	$885,000
3 (low)	$860,000
4	$877,500
5	$875,000

10. What is the land value range for the subject?
    a. Between $210,000 and $227,000
    b. Between $222,300 and $222,500
    c. Neither a nor b
    d. Cannot be determined

LESSON 2 REVIEW ANSWERS

1. d
2. b
3. c
4. b; $600,000 ÷ 7
5. c; value of one part × number of building parts = $85,714 × 6
6. b; one part ÷ total number of parts = 1 ÷ 7
7. c; six parts ÷ total number of parts = 6 ÷ 7
8. d; one part ÷ total number of parts = 1 ÷ 6
9. a; value of one part × number of building parts = $83,333 × 5
10. b

    First, determine the typical land sales price:

    $225,000 + $210,000 + $227,000 = $662,000

    $662,000 ÷ 3 = $220,667 = $221,000 rounded

Because the improved properties in a similar neighborhood have generally sold from $860,000 to $885,000, the land value would range from approximately 25 percent to 26 percent of total property value ($221,000 ÷ $885,000 and $221,000 ÷ $860,000).

Because improved properties in the same neighborhood as the subject are selling for between $855,000 and $890,000, the land value range for the subject would range from $222,300 to $222,500 (0.25 × $890,000 and 0.26 × $855,000).

LESSON 3: Extraction or Abstraction Method

When the sales price of a comparable property is known, the depreciated improvement costs of the comparable can be subtracted to arrive at a land value. This technique of land or site valuation is known as the **extraction method** or **abstraction method.** The extraction method utilizes the following methodized process of the cost approach (discussed in Section 3):

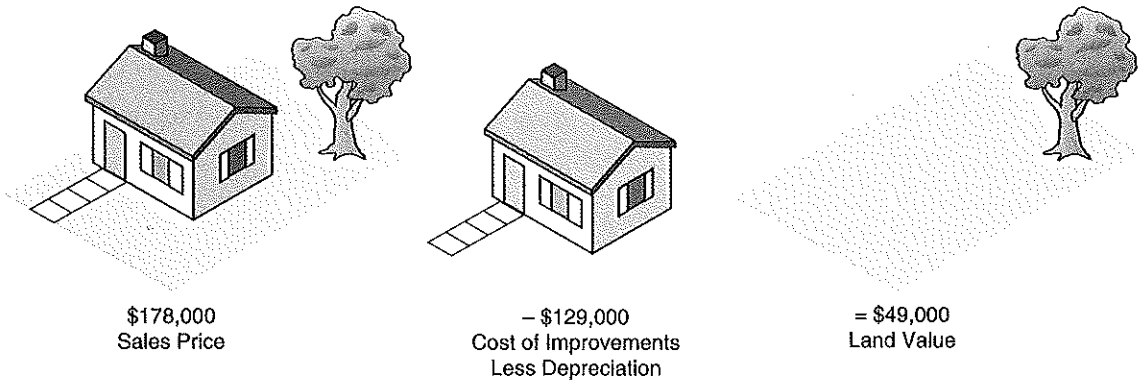
	Replacement or reproduction cost new (RCN)
–	Accrued depreciation
=	Depreciated cost of the improvements
+	Land (site) value
=	Indicated value

The indicated value is the result of adding the depreciated cost of the improvements to the land (site) value.

In order to use this technique of land or site valuation, the following steps should be taken:

1. Obtain sales prices of comparable properties located within subject or similar neighborhood.
2. Calculate an estimate of the comparable's RCN as of the date of the sale.

FIGURE 2.2
Extraction Method



3. Estimate the accrued depreciation of the comparable sale as of the date of sale.
4. Subtract the estimated accrued depreciation (step 3) from the RCN of the comparable (step 2). This results in the depreciated cost of the improvements.
5. Subtract the depreciated cost of the improvements (step 4) from the total sales price of the comparable (step 1). This results in the indicated land/site value of the comparable.
6. Compare the indicated land/site value of the comparable (step 5) to the subject site and adjust for any differences.

Steps 1–5 simply rearrange the basic cost approach formula in order to isolate land or site value as follows:

	Replacement or reproduction cost new (RCN)
–	Accrued depreciation
=	Depreciated cost of the improvements

	Indicated value (sales price)
–	Depreciated cost of the improvements
=	**Land (site) value**

For example, a comparable property sold for $360,000. The replacement cost new (RCN) is estimated at $450,000 and the estimated accrued depreciation is 32 percent as of the date of sale. What is the estimate of land/site value?

Using the rearranged basic cost approach formula:

	$450,000
–	144,000*
	$306,000

	$360,000
–	306,000
=	**$ 54,000**

*$144,000 represents the accrued depreciation calculated by multiplying the accrued depreciation percentage by the RCN ($450,000 × 0.32).

In Practice

There are no comparable vacant land sales within the subject's neighborhood. However, there are comparable sales of *improved* properties similar to the subject. Accrued depreciation for improved properties similar to the subject in age, size, condition, and appeal is estimated at 35 percent. The following grid shows the estimated cost of improvements for all six comparables using the cost approach. Use the extraction method to estimate *typical* land/site value as well as the *typical* land/site percentage by using the following table:

Sale	Sales Price	Cost of Improvements (RCN)	Depreciated Cost of Improvements	Land Value	Land Value Percentage
1	$440,000	$550,000			
2	$442,000	$545,000			
3	$430,000	$520,000			
4	$439,000	$525,000			
5	$450,000	$560,000			
6	$455,000	$560,000			

Solution:

Sale	Sales Price	Cost of Improvements (RCN)	Depreciated Cost of Improvements	Land Value	Land Value Percentage
1	$440,000	$550,000	$357,500	$102,500	22.3%
2	$442,000	$545,000	$354,250	$107,750	23.3%
3	$430,000	$520,000	$338,000	$102,000	23.2%
4	$439,000	$525,000	$341,250	$97,750	22.3%
5	$450,000	$560,000	$364,000	$106,000	22.6%
6	$455,000	$560,000	$364,000	$111,000	23.4%
Mean				$104,500	22.8%

*The depreciated cost of the improvements is calculated by deducting the estimated accrued depreciation percentage (35%) from the RCN. Land value is calculated by subtracting depreciated cost of improvements from the sales price.

The typical land/site value is $104,500 and the typical land/site percentage is 22.8 percent.

LESSON 3 REVIEW QUESTIONS

1. The extraction method is also known as the
   a. allotment method.
   b. allocation method.
   c. abstraction method.
   d. ratio method.

2. The extraction method is used when
   a. the sales price of a comparable property is known.
   b. the depreciated improvement costs of the comparable are estimated.
   c. Both a and b
   d. None of the above

3. The extraction method of valuing land or site includes
   a. obtaining sales prices of comparable properties located within the subject or similar neighborhood.
   b. calculating an estimate of the comparable's RCN as of the date of the sale.
   c. estimating the accrued depreciation of the comparable sale as of the date of sale.
   d. All of the above

4. Which of the following statements is true?
   a. Depreciated cost of the improvements = RCN – Accrued deprecation
   b. Land value = Sales price – Depreciated cost of the improvements
   c. Depreciated cost of the improvements – Accrued depreciation = RCN
   d. All of the above

5. A comparable property sold for $775,000. The replacement cost new (RCN) is estimated at $925,000 and the estimated accrued depreciation is 29 percent as of the date of sale. What is the estimate of land/site value?
   a. $268,250
   b. $656,250
   c. $118,250
   d. $150,000

Answer questions 6–9 using the following information:

A comparable property sold for $425,000. The replacement cost new (RCN) is estimated at $550,000 and the estimated accrued depreciation is 42 percent as of the date of sale.

6. What is the accrued depreciation?

7. What is the depreciated cost of the improvements?

8. What is the estimate of land/site value?

9. What is the land or site value percentage?

10. A comparable new construction property sold for $680,000. The replacement cost new (RCN) is estimated at $475,000 and the estimated accrued depreciation is 0 percent as of the date of sale. What is the estimated site value?
    a. $205,000
    b. $475,000
    c. $680,000
    d. Cannot be determined

LESSON 3 REVIEW ANSWERS

1. c

2. c

3. d

4. d

5. c

$$\begin{array}{r} \$925{,}000 \\ -\ 268{,}250 \\ \hline \$656{,}750 \end{array}$$

$$\begin{array}{r} \$775{,}000 \\ -\ 656{,}750 \\ \hline = \ \mathbf{\$118{,}250} \end{array}$$

6. $231,000 ($550,000 × 0.42)

7. $319,000 ($550,000 – $231,000)
8. $106,000 ($425,000 – $319,000)
9. 24.9% ($106,000 ÷ $425,000)
10. a

$$\begin{array}{r} \$475{,}000 \\ - \quad 0 \\ \hline \$475{,}000 \end{array}$$

$$\begin{array}{r} \$680{,}000 \\ - \quad 475{,}000 \\ \hline = \$205{,}000 \end{array}$$

LESSON 4: Subdivision Development Method

With the **subdivision development method**, the costs of developing and subdividing a parcel of land can be subtracted from the expected sales prices of the subdivided sites. Adjustments are made for time needed to sell all subdivided sites in order to derive the value of the undivided raw land.

This method is used primarily to value land that is in transition from one use to another where direct sales of similar uses are not available. The subdivision development method theoretically matches the thinking process in which a developer would engage when considering a higher and better use of the land.

For example, assuming that the highest and best use for 60 acres of raw land is to develop it into a subdivision of 80 residential lots, the total project cost including developer's profit is $5,000,000. The appraiser estimates the lots will sell for $110,000 each for a total of $8,800,000. The raw land is then valued at $3,800,000 ($8,800,000 – $5,000,000) or $47,500 per residential lot. This theoretical value is based on the recapture (recovery) of the costs including the profits. The result is consistent with what a developer should be willing to pay for the land if he could sell all of the lots at once. Because the lots will not sell all at once, it is necessary for this theoretical value to be discounted to a present value.

The principle of anticipation is fundamental to the subdivision development method, because the basis for this method is the present value for a developer to undertake a project. According to Fisher, Martin, and Mueller (*The Language of Real Estate Appraisal*), the process of analysis under this method is a follows: "...the number and size of lots that can be economically, legally, and physically created is first analyzed. Then comparable sales of finished lots are used to develop an absorption rate, development period, and total estimated sales price. Next, income and expenses are forecasted. Finally, the net cash flows are discounted to a present value in order to estimate the value of the land."

The **absorption rate** refers to the amount of time over which it is estimated that the individual lots will sell. **Discounting** is the financial concept of converting an estimated future income into a present value using a discount rate. Discounting an expected future cash flow into a present value bases on the concept that the rights associated with receiving the cash flow in the future is less valuable than receiving the income today. Simply stated, it is the present value of a future benefit. For a complete discussion on the time-value of money and present value factors (present value of $1) see Appendix, Lesson 4.

For example, 80 acres of raw land is to be developed into a subdivision of 100 residential lots. The total project cost including developer's profit is $7,500,000. The appraiser estimates the lots will sell for $125,000 each for a total of $12,500,000. If it is expected that the absorption of the project will take four years (25 lots per year), and that the discount rate is estimated to be 10 percent per year, what is the present value of the raw land?

Because the lots will not sell all at once, it is necessary for the theoretical land value to be discounted. The present value factors for the four years at 10 percent are:

Year 1 = 0.909090

Year 2 = 0.826446

Year 3 = 0.751314

Year 4 = 0.683013

First, calculate the theoretical raw land value:

$5,000,000 ($12,500,000 – $7,500,000) or $50,000 per lot

Next, because it will take four years to sell off at 25 lots per year, this translates into a cash flow of:

$50,000 × 25 = $1,250,000 per year

Year 1	$1,250,000 × 0.909090 =	$1,136,363
Year 2	$1,136,363 × 0.826446 =	$939,143
Year 3	$939,143 × 0.751314 =	$705,591
Year 4	$705,591 × 0.683013 =	$481,928
Present value total		**$3,263,025**

$3,263,023 ÷ 80 = $40,788 per acre raw land

Therefore, the developer is justified in paying $40,788 per acre. This is an oversimplified example because the developer's costs and revenues will more than likely fluctuate during each absorption period.

LESSON 5: Ground Rent Capitalization Method

The **ground rent capitalization method** involves a **ground lease,** which refers to the agreement between a landowner and a tenant to erect a building on the site. Ground leases are usually long term (50 years or more). The tenant in a long-term lease wants the lease to run as long as the estimated useful life of the building. Occasionally, the owner of land used for commercial or other purposes decides to retain the fee simple title to the land yet lease the building erected on the site. The income associated with the ground lease is then capitalized using the IRV formula to derive a land value. (See Figure 2.3.)

For example, if the ground lease generates an annual income of $60,000 and the market reflects a capitalization rate of 8 percent, the value using the IRV formula is:

Value (V) = Income (I) ÷ Capitalization rate (R)

= $60,000 ÷ 0.08

= $750,000

■ **In Practice**

If a ground lease generates a monthly income of $35,000 and the market reflects a capitalization rate of 10 percent, what is the value using the IRV formula?

Solution:

Value (V) = Income (I) ÷ Capitalization rate (R)

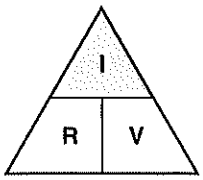
= ($35,000 × 12 months) ÷ 0.10

= $420,000 ÷ 0.10

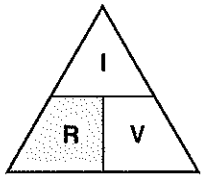
= $4,200,000

The appraiser must know the quantity, quality, and durability of the ground lease. This method can be used when there is an active market for ground leases and a direct capitalization rate can be derived from the market. In other words, the appraiser knows the income derived from the ground leases and the sales prices of comparable sales.

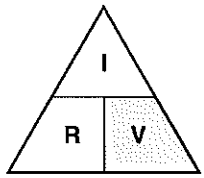
FIGURE 2.3
Direct Capitalization Relationships



$I = R \times V$



$R = I \div V$



$V = I \div R$

LESSON 5 REVIEW QUESTIONS

1. Which of the following statements is true?
   a. The ground rent capitalization method can be used when there is an active market for ground leases and a direct capitalization rate can be derived from the market.
   b. The appraiser must know the quantity, quality, and durability of the ground lease.
   c. Both a and b
   d. Neither a nor b

2. Using the IRV formula, value is calculated by
   a. dividing the income by the capitalization rate [Income (I) ÷ Capitalization rate (R)].
   b. multiplying the income by the capitalization rate [Income (I) × Capitalization rate (R)].
   c. dividing the capitalization rate by the income [Capitalization rate (R) ÷ Income (I)].
   d. All of the above

3. If a ground lease generates an annual net income of $25,000 and the market reflects a capitalization rate of 9 percent, the value using the IRV formula is
   a. $277,777.
   b. $3,333,333.
   c. $1,500,000.
   d. $259,000.

4. If a ground lease generates a monthly net income of $6,000 and the market reflects a capitalization rate of 7 percent, the value using the IRV formula is
   a. $85,715.
   b. $80,029.
   c. $72,000.
   d. $1,028,571.

5. If a ground lease generates a quarterly net income of $5,000 and the market reflects a capitalization rate of 8 percent, the value using the IRV formula is
   a. $750,000.
   b. $60,000.
   c. $250,000.
   d. $62,500.

LESSON 5 REVIEW ANSWERS

1. c
2. a
3. a ($25,000 ÷ 0.09)
4. d ($6,000 × 12) ÷ 0.07
5. c ($5,000 × 4) ÷ 0.08

LESSON 6 REVIEW QUESTIONS

1. Which of the following statements is true about the land residual method of land/site valuation?
   a. The net income from the building is deducted from the total net income derived from the entire property (land and building). The balance is then net income derived from just the land, which is then capitalized using the IRV formula.
   b. By applying the IRV formula to the building capitalization rate and the value of the building, only the net income attributed to the building can be determined.
   c. Both a and b
   d. Neither a nor b

2. In order to value land using the land residual method, what needs to be known about the property?
   a. The value (depreciated cost) of the building (V_B)
   b. The market capitalization rate for the building (R_B) and the land (R_L)
   c. The total property (land and building) net income or income overall (I_O)
   d. All of the above

3. A commercial site is to be developed with a 2,500-square-foot retail building. The cost to construct the building is $170 per square foot. The market-derived capitalization rate for land and building is 7 percent and 10 percent, respectively. The annual net operating income attributed to the entire property (land and building) is an estimated $60,000. What is the value of the land using the land residual method?
   a. $17,500
   b. $250,000
   c. $425,000
   d. $60,000

4. A commercial site is to be developed with a 2,800-square-foot industrial building. The cost to construct the building is $200 per square foot. The market-derived capitalization rate for land and building is 7 percent and 10 percent, respectively. The annual net operating income attributed to the entire property (land and building) is an estimated $80,000. What is the value of the land using the land residual method?
   a. $56,000
   b. $560,000
   c. $24,000
   d. $342,857

5. Using the information in question 4, what is the building income (I_B)?
   a. $56,000
   b. $560,000
   c. $24,000
   d. $342,857

LESSON 6 REVIEW ANSWERS

1. c

2. d

3. b

$$\text{Income } (I_B) = \text{Value } (V_B) \times \text{Capitalization rate } (R_B)$$

$$= (\$170 \times 2{,}500) \times 0.10$$

$$= \$425{,}000 \times 0.10$$

$$= \$42{,}500\ (I_B)$$

$$\text{Land income } (I_L) = \text{Total property income } (I_O) - \text{Building income } (I_B)$$

$$\$17{,}500 = \$60{,}000 - \$42{,}500$$

$$\text{Value } (V_L) = \text{Income } (I_L) \div \text{Capitalization rate } (R_L)$$

$$= \$17{,}500 \div 0.07$$

$$= \$250{,}000\ (V_L)$$

4. d

$$\text{Income } (I_B) = \text{Value } (V_B) \times \text{Capitalization rate } (R_B)$$

$$= (\$200 \times 2{,}800) \times 0.10$$

$$= \$560{,}000 \times 0.10$$

$$= \$56{,}000\ (I_B)$$

Land income (I_L) = Total property income (I_O) – Building income (I_B)

$24,000 = $80,000 – $56,000

Value (V_I) = Income (I_I) ÷ Capitalization rate (R_I)

= $24,000 ÷ 0.07

= $342,857 ($V_L$)

5. a

Income (I_B) = Value (V_B) × Capitalization rate (R_B)

= ($200 × 2,800) × 0.10

= $560,000 × 0.10

= $56,000 ($I_B$)

SECTION 2 REVIEW QUESTIONS

1. Which land/site valuation techniques use the IRV formula?
   a. All six methods of land/site valuation
   b. The ground rent capitalization method and the land residual method
   c. The sales comparison approach and the allocation method
   d. The allocation method and the abstraction method

2. Which of the following statements is true about the ground rent capitalization method?
   a. The net income associated with the ground lease is capitalized using the IRV formula to derive a land value.
   b. This method can be used when there is an active market for ground leases and a direct capitalization rate can be derived from the market.
   c. The tenant in a long-term lease will want the lease to run as long as the estimated useful life of the building.
   d. All of the above

3. Which order should percentage adjustments be made in?
   a. Financing, property rights conveyed, conditions of sale, market conditions (time)
   b. Property rights conveyed, conditions of sale, financing, market conditions (time)
   c. Financing, property rights conveyed, market conditions (time), conditions of sale
   d. Property rights conveyed, financing, conditions of sale, market conditions (time)

4. A unit of comparison is calculated by the formula
   a. sales price ÷ element of comparison.
   b. element of comparison ÷ sales price.
   c. Both a and b
   d. None of the above

5. If the typical land to building ratio is 1:6, then land is
   a. ⅐ of the total building value.
   b. ⅐ of the total property value.
   c. ⅙ of the total property value.
   d. ⅙ of the total building value.

section three

III

THE COST APPROACH

LESSON 1: Concepts and Definitions

LESSON 2: Methods of Cost Analysis

LESSON 3: Depreciation

LEARNING OBJECTIVES

By the end of this section, participants will be able to

- identify the steps involved with the cost approach;
- perform all calculations within the cost approach;
- understand the process of the cost approach;
- comprehend the strengths and limitations of the cost approach;
- understand reconciliation within the cost approach;
- recognize the various methods of costing;
- distinguish between depreciation and obsolescence;
- understand curable and incurable items; and
- understand the principles related to the cost approach.

KEY TERMS

accrued depreciation
actual age
breakdown method
chronological age
cost approach
cost index method
curable
curable functional obsolescence
curable physical depreciation
depreciation
direct costs
economic age-life method
effective age (EA)
engineering breakdown method
entrepreneurial profit
external obsolescence
functional obsolescence
incurable
incurable functional obsolescence
incurable physical depreciation
indirect costs
long-lived item
market method
modified economic age-life method
obsolescence
physical depreciation
quantity survey method
remaining economic life (REL)
replacement cost new (RCN)
reproduction cost new
short-lived item
square-foot method
straight-line method
superadequacy
total economic life (TEL)
unit-in-place method

LESSON 1: Concepts and Definitions

The **cost approach** is a methodized process to derive a value indication whereby the *replacement cost*, or the *reproduction cost*, of a building is first estimated including the *entrepreneurial profit*, then *accrued depreciation* is subtracted, and finally the value of the land or site as though vacant is added to conclude a value indication.

RELATED PRINCIPLES

There are several appraisal principles related to the cost approach that are worth mentioning. These principles explain how markets work and they describe the interaction of the forces within markets. Therefore, it is certainly worth noting how these principles reflect the actual figures involved with the market's decision making processes.

For example, the principle of *substitution* has applicability within all of the approaches to value, and therefore a slightly different interpretation can be expected within its application relative to each approach. The principle of substitution is the principle that is primary to the cost approach. The principle of substitution is clearly demonstrated in a market comparison (sales comparison) situation whereby the principle states that all things being equal between two items of similar utility and demand, the lower priced one will sell first. This is the classic margarine versus butter problem. If the price of butter becomes too high, eventually people will substitute butter with a similar product of like utility, in this case margarine. Likewise, if two properties of similar utility and demand are on the market, the lower priced property will likely sell first. Of course, this assumes a knowledgeable buyer acting in his or her own best interests and so on. The principle of substitution as it applies to the cost approach has a slightly different interpretation and therefore a slightly different application.

The principle of substitution as it applies to the cost approach states that "no person is justified in paying more for a property than they can build new." This of course assumes that there is no burden of undue delay; in other words, that the person building the property is not in need of the property before the time it takes to build. To put it in simpler terms, the principle of substitution under the cost approach asks: "If I can wait for the time it takes to build the property and assemble all of the agents of production, why should I pay more for an existing property of similar utility?"

The answer is, "I shouldn't." Therefore, you see there is a slightly different interpretation of the principle of substitution as it relates to the cost approach than the sales comparison (market) approach.

Implied in these principles and forces are other principles such as *supply and demand* and *the four agents of production* (land, labor, capital, and management), and because supply and demand are noted, then so too must the principles of *balance* and *competition*. A student might do well here to review the principles of real estate.

Supply and demand have a correlated effect upon most markets and the interrelationship of these two forces can have a predictable effect on value. However, within a market there are also additional complex forces at work that might make such predictions difficult or impossible to project without further data. These additional complexities are what make a market an imperfect entity—and most markets, in particular real estate, are imperfect markets. While all of these appraisal principles are a solid foundation for explaining or interpreting a market, they are not the forces themselves. Such forces can only be isolated with market analysis and solid market data that is applied and interpreted under the approaches to value.

As an example, the principle of externality is an exact interpretation of the concept of *external obsolescence*, and the basis of a portion of accrued depreciation in the cost approach. You will remember that under the principle of externality, it is recognized that the influences outside of the property lines might have an effect on the property.

The principle of *contribution* applies to the cost approach by supporting the idea that each component of the cost approach has a contribution to the overall value. The principle of *surplus productivity* states that all addition to value after the agents of production have been satisfied will be applied to the land. Therefore the principle of surplus productivity speaks to the land value portion of the cost approach. In addition, of course, one could not complete a highest and best use analysis without the cost approach to isolate financial feasibility, or the value of the land as if vacant.

METHODIZED PROCESS OF THE COST APPROACH

	Replacement or reproduction cost new (RCN)
–	Accrued depreciation
=	Depreciated cost of the improvements
+	Land (site) value as though vacant
=	Indicated value using the cost approach

RCN (Replacement Cost New/Reproduction Cost New)

Replacement cost is the cost of constructing new the existing building at today's construction standards using today's materials. This is also known as **replacement cost new** and is the beginning point of a cost analysis using replacement cost figures for an existing building. *Reproduction cost* is the cost of reproducing an exact duplicate of the existing building using the exact materials and the construction standards as the original construction. This is also known as **reproduction cost new,** and is the beginning point of a cost analysis using reproduction cost figures for an existing building.

The main difference between replacement and reproduction is that replacement cost estimates eliminate superadequacy or other functional obsolescence associated with the existing improvement. Reproduction cost might include obsolescence because it is an exact duplicate of the original. A *special-use* property, such as a historic building, a church, school, library, or other unique property, is the typical application for the reproduction cost new.

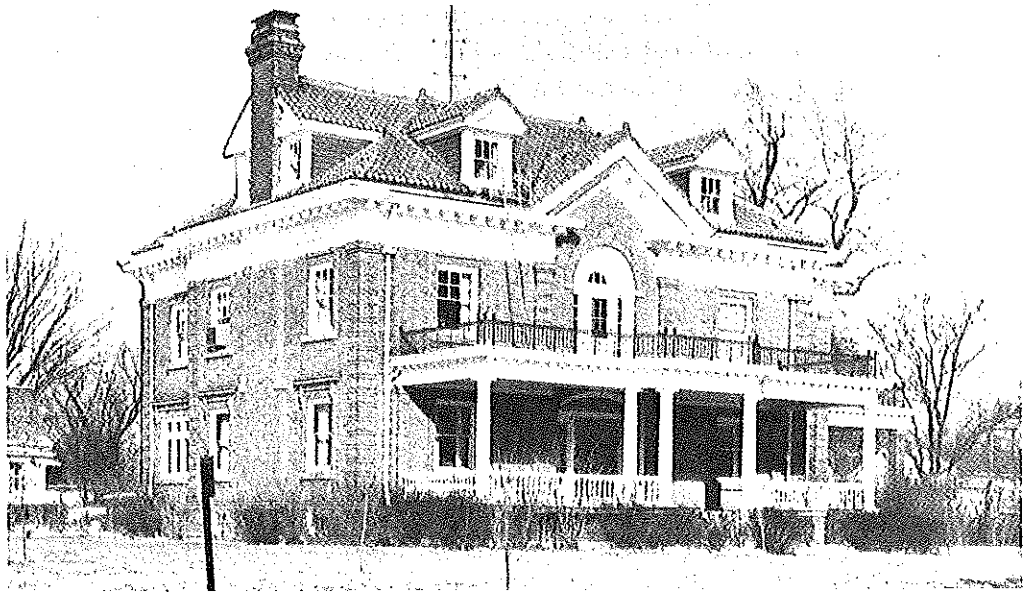
The construction cost—whether replacement or reproduction—will be for a new structure as of the effective date (the date at which the analysis and opinions apply).

The home at the bottom of Figure 3.1 is a typical use for the replacement cost new. Replacement cost new is used because it would not be economically feasible to reproduce a home of this age with the same quality, materials, and artisanship of the past.

Applicability

The applicability of the cost approach is founded in the principle of substitution as mentioned above under "related principles." Remember that the principle of substitution related to the cost approach stated that "no person is justified paying more for a property than they could build new without undue delay. The implication of the related principle of substitution is that market value and the cost of the property are closely related with newer properties. This leads to the concept that the cost approach is likely to be most "applicable" in newly constructed or relatively new properties. In theory, the market is likely to be "knowledgeable" about the forces that affect and create value and therefore the property would not be built if the cost to build was not feasible. Given that the land value is well-supported

FIGURE 3.1
Replacement Cost



and that the costs associated with the construction of the property are similar to the current cost new with little or no accrued depreciation, the cost approach has substantial validity as an indicator of value.

The cost approach also is useful for determining market value in situations where the sales comparison and income approaches lack validity or applicability. For example, in single-family valuation the income approach may not have validity in a market where the income stream associated with the ownership of property does not drive ownership of the property, and the

market might lack recent or comparable sales that would support a value conclusion under the sales comparison approach. Likewise, if the property has a low "effective age," and therefore little physical depreciation with identifiable or little obsolescence, then the cost approach is likely to be representative of the market value of the property.

Another situation where the cost approach is helpful is when a property is to be considered with proposed renovations or additions. The cost approach will point to overimprovements or superadequacies. Finally, the cost approach has use when the assignment calls for feasibility consideration. If the cost of completing proposed changes is less than the value return associated with the changes, then the project is considered "financially feasible." The cost approach is therefore instrumental in feasibility analysis as the very definition of feasibility is based on the question of the value return associated with the cost of a proposed change.

Limitations

So it goes that the cost approach, like the other approaches, has limitations in applicability or validity. Often, the limitations are found in the data extracted from the market. Some obvious limitations are the lack of accurate vacant land sales, or excessive depreciation, leaving the accuracy of the cost approach to be questionable. Also, the use of a property within a market may limit the effectiveness of the cost approach. Although cost approach is typically used with special use property (libraries, churches, etc.), a nonconforming use property, or a building that is not put to its highest and best use, there may be limitations to its use. For example, if the sales comparison and income approach readily reflect available and reliable data that can support market value, then the cost approach is not likely to be truly representative of the market value of the property. If this is the case, the cost is not likely to match the market response of such properties because of their unique "niche" in the market.

LESSON 1 REVIEW QUESTIONS

1. The cost approach is a methodized process to derive a value indication whereby the _____, or the _____, of a building is first estimated.

2. No person is justified in paying more for a property than they can build _____.

3. _____ cost is the cost of constructing an exact duplicate of the existing building using the exact materials and the construction standards of the original building.

4. Replacement cost is the cost of constructing new the existing building at _____ construction standards using _____ materials.

5. The cost approach has applicability and _____ depending on the situation and data available.

6. The principle of _____ is the primary principle of the cost approach.

7. A special-use property is a typical use of the _____ cost.

8. Another situation where the cost approach is helpful is when a property is to be considered with proposed _____ or additions.

9. If the property has a low "effective age," it is likely to have little _____ _____ depreciation.

10. If the cost of completing proposed changes is less than the value return associated with the changes, then the project is considered _____ _____.

LESSON 1 REVIEW ANSWERS

1. replacement cost; reproduction cost

2. new

3. Reproduction

4. today's; today's

5. limitations
6. substitution
7. reproduction
8. renovations
9. physical
10. financially feasible

LESSON 2: Methods of Cost Analysis

There are four methods for estimating the construction costs whether replacement or reproduction: the *cost index method*, the *square-foot method*, the *unit-in-place method*, and the *quantity survey method*. This lesson explains each of these methods.

COST INDEX METHOD

The **cost index method** estimates construction cost by multiplying the known original cost of a building by a factor. Cost index or cost reporting services keep records of all building costs over time. The factor represents the percentage increase in construction costs up to the effective date of value. This factor is then applied to the original cost of the subject building. The formula used is:

$$\frac{\text{Present index}}{\text{Index at time of construction}} \times \text{Original cost} = \text{Present cost}$$

For example, a house was built for $100,000 in 1986. At that time, the construction cost index was 120. Currently (as of the effective date of the appraisal), the cost index is reported at 300. Calculate the present cost as follows:

$$\frac{300}{120} \times \$100{,}000 = \$250{,}000$$

■ In Practice

The original construction cost of a building was $130,000, with a cost index of 175. What are the current estimated construction costs for the building if the cost index is now 290?

Solution:

Using the cost index formula:

Present index = 290

Index at time of construction = 175

Original cost = $130,000

$$\frac{290}{175} \times \$130{,}000 = \$215{,}429$$

This method is the least reliable when used alone but can be used to support or verify cost figures derived from the other methods. Although original construction costs may be available, it is difficult to determine if these costs represent what would be "typical" costs at that time.

SQUARE-FOOT METHOD

The **square-foot method** breaks the structure down into a square-foot unit of comparison. The appraiser can either collect cost data of a comparable new structure in the area or refer to published cost manuals. Because construction costs may vary in direct proportion to the size of the building (economies of scale), it is important that the appraiser chooses a comparable sale that is similar in size to the subject. Cost manuals indicate typical constructions, which are usually updated quarterly or monthly. The quality of the construction and material is first rated with similar construction costs, and then the cost per square foot is adjusted for differences in upgrades, or options. Sometimes a local factor is required for regional differences as a result of labor, shipping, or access costs due to difficult terrain. Figure 3.2 shows a page from a typical residential cost manual. The total cost for this house is estimated at $62.55 per square foot. This figure can then be adjusted for regional differences. The square-foot method is the most used method for calculating building costs.

■ In Practice

You are appraising a single-family home with a total area of 3,100 square feet. The cost to build a local comparable 3,300-square-foot residence is $415,000.

Using construction costs from the comparable single-family home, estimate the construction costs of the subject residence.

Solution:

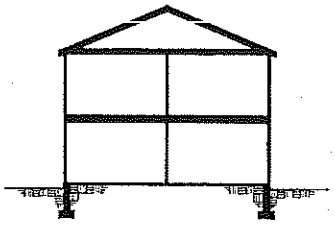
Cost per square foot of comparable residence:

$415,000 ÷ 3,300 = $125.76 per sq. ft.

Cost estimate of subject:

3,100 × $125.76 = $389,856

FIGURE 3.2
Sample Page from a Residential Construction Cost Manual



Economy 2 Story

Living Area - 2000 S.F.
Perimeter - 135 L.F.

			Labor Hours	Cost Per Square Foot Of Living Area		
				Mat.	Labor	Total
1	Site Work	Site preparation for slab; 4' deep trench excavation for foundation wall.	0.034		0.59	0.59
2	Foundation	Continuous reinforced concrete footing, 8" deep x 18" wide; dampproofed and insulated 8" thick reinforced concrete block foundation wall, 4' deep; 4" concrete slab on 4" crushed stone base and polyethylene vapor barrier, trowel finish	0.069	2.27	3.04	5.31
3	Framing	Exterior walls - 2" x 4" wood studs, 16" O.C.; 1/2" insulation board sheathing; wood truss roof frame, 24" O.C. with 1/2" plywood sheathing, 4 in 12 pitch; 2" x 8" floor joists 16" O.C. with bridging and 5/8" plywood subfloor.	0.112	4.70	5.63	10.33
4	Exterior Walls	Beveled wood siding and #15 felt building paper on insulated wood frame walls; 6" attic insulation; double hung windows; 2 flush solid core wood exterior doors with storms.	0.107	7.50	4.26	11.76
5	Roofing	20 year asphalt shingles; #15 felt building paper; aluminum gutters, downspouts, drip edge and flashings.	0.024	0.47	0.91	1.38
6	Interiors	Walls and ceilings, 1/2" taped and finished drywall, primed and painted with 2 coats; painted baseboard and trim; rubber backed carpeting 80%, asphalt tile 20%; hollow core wood interior doors.	0.219	7.73	10.17	17.90
7	Specialties	Economy grade kitchen cabinets - 6 L.F. wall and base with plastic laminate counter top and kitchen sink; 30 gallon electric water heater.	0.017	0.83	0.43	1.26
8	Mechanical	1 lavatory, white, wall hung; 1 water closet, white; 1 bathtub, enameled steel, white; gas fired warm air heat.	0.061	2.12	2.17	4.29
9	Electrical	100 Amp. service; romex wiring; incandescent lighting fixtures; switches, receptacles.	0.030	0.58	0.96	1.54
10	Overhead	Contractor's overhead and profit		3.95	4.24	8.19
			Total	30.15	32.40	62.55

Source: Means Residential Cost Data 2005. Copyright Reed Construction Data, Kingston, MA 781-585-7880. All rights reserved.

UNIT-IN-PLACE METHOD

The **unit-in-place method**, also known as *component costing* or the *segregated cost method*, calculates the improvement costs by analyzing the costs for major components of a structure. Detailed costs are provided from cost manuals or costing services, as well as from local builders and suppliers. These costs include the following building components:

- Appliances
- Ceiling
- Countertops
- Cabinets
- Electrical
- Electrical fixtures
- Exterior walls
- Fireplace(s)
- Floor coverings
- Floor structure
- Foundation
- Heating and cooling system
- Interior construction
- Plumbing
- Plumbing fixtures
- Roof dormers
- Roof structure and covering
- Stairway(s)

Adding together the major components concludes a total cost new of the improvements. Figure 3.3 is an example of costs for an industrial building obtained from a cost manual.

FIGURE 3.3
Cost Data

Component	Cost per Measured Unit
Foundation 12" concrete wall and footings	$30.70 per linear foot
Floor Construction 8" reinforced concrete	$3.60 per sq. ft. of floor area
Framing 14' steel columns, beams, and purlins	$4.50 per sq. ft. of support area
Roof Construction sheathing, 2" polystyrene insulation, 4-ply asphalt and gravel covering	$3.70 per sq. ft.
Exterior Walls 12" concrete block backup	$10.20 per sq. ft.
Windows industrial sash, steel, 50% vented	$14.20 per sq. ft.
Doors hollow metal, 3' × 7' rolling steel, chainhoist operated, 12' × 12'	$355 per door $1,425 per door
Interior Painting	$0.35 per sq. ft.
Electrical Wiring and Fixtures	$3.10 per sq. ft. of building area
Heating and A/C	$5.30 per sq. ft. of building area
Plumbing including Fixtures	$2.25 per sq. ft. of building area
Parking Area 3" asphalt on 3" stone base	$7.20 per sq. yd.

In Practice

Using the following information about a rectangular warehouse, calculate the cost to construct using the unit-in-place method.

Building dimensions: 100' × 275', building area = 27,500 sq. ft.

Foundation: concrete walls and footing at $41.20 per linear foot

Floor: reinforced concrete at $3.75 per sq. ft.

Roof: built-up tar and gravel at $2.75 per sq. ft.;

roof sheathing at $0.75 per sq. ft.;

fiberboard insulation at $0.65 per sq. ft.

Interior construction: partition walls, interior painting: total cost $5,100

Front exterior wall:	common brick on concrete block, 100' × 15' high at $10.25 per sq. ft.;
	two windows each 6' × 12' at $16.10 per sq. ft.;
	one 10' × 12' overhead door at a cost of $1,400; one 3' × 7' hollow core metal door at $375
Side exterior walls:	concrete block 275' long × 15' high at $8.50 per sq. ft.;
	windows covering 20% of wall area at $15.25 per sq. ft.
Rear exterior wall:	concrete block 100' × 15' at $8.50 per sq. ft.;
	one 10' × 12' overhead door at a cost of $1,400
Steel framing:	area supported by frame: 75' × 250' with 14" eave height, at $5.10 per sq. ft.
Electrical:	$3.75 per sq. ft. of floor area
Heating:	$2.80 per sq. ft. of floor area
Plumbing:	$2.00 per sq. ft. of floor area

Solution:

Unit-in-place costs:

Foundation:

Perimeter = (2 × 100) + (2 × 275) = 750 ft.

750 feet @ $41.20	$30,900

Floor:

27,500 sq. ft. @ $3.75	$103,125

Roof:

27,500 sq. ft. @ $4.15 ($2.75 + $0.75 + $0.65)	$114,125

Interior construction:

Partition walls, interior painting:	$5,100

Front exterior wall:

100' × 15' = 1,500 sq. ft.

a. Windows: 144 sq. ft. (72 sq. ft. × 2)

b. Overhead door: 120 sq. ft.

c. Door: 21 sq. ft.

1,215 sq. ft. @ $10.25	$12,454	
Windows: 144 sq. ft. @ $16.10	2,318	
Overhead door: $1,400	1,400	
Door: $375	375	
Total		$16,547

Side exterior walls:

2 × (275' × 15') = 8,250 sq. ft. – 1,650 sq. ft. (20% for windows)

= 6,600 sq. ft. @$8.50	$56,100	
1,650 sq. ft. @ $15.25	25,163	
Total		$81,263

Rear exterior wall:

100' × 15' = 1,500 sq. ft.

d. Overhead door: 120 sq. ft. (1,500 – 120 = 1,380)

1,380 sq. ft. @ $8.50	$11,730	
Overhead door: $1,400	1,400	
Total		$13,130

Steel framing:

75' × 250' = 18,750 sq. ft. @ $5.10	$95,625

Electrical:

27,500 sq. ft. @ $3.75	$103,125

Heating:	
27,500 sq. ft. @ $2.80	$77,000
Plumbing:	
27,500 sq. ft. @ $2.00	$55,000
Total replacement/reproduction cost	**$694,940**

Total cost per sq. ft. = **$25.27** ($694,940 ÷ 27,500)

QUANTITY SURVEY METHOD

The **quantity survey method,** sometimes called the *builder's breakdown method,* is the most comprehensive method of costing. It is similar to the unit-in-place method; however, the costs are estimated by accounting for every item in a project, including the cost of individual units of materials, labor, fees, and so on. The appraiser itemizes these costs separately and places them into categories. With the quantity survey method, *direct costs, indirect costs,* and *entrepreneurial profit* are computed more precisely than in any other method. **Direct costs,** also referred to as *hard costs,* include expenditures for labor and materials used in the construction of an improvement including the contractor's overhead and profit. **Indirect costs,** also referred to as *soft costs,* include financing costs, construction loan interest, insurance and real estate taxes during construction, building permits, administrative costs, professional fees (appraisal, engineering, and architectural fees), and lease-up and marketing costs involved with stabilizing occupancy.

Entrepreneurial profit, also known as developers' profit, is the difference between the price paid for a new property and the total cost to deliver the property. This includes the direct and indirect costs plus land value. Entrepreneurial profit is only realized when the improved property sells; therefore, it is an estimate of anticipated profit to the entrepreneur. Published cost manuals do *not* include entrepreneurial profit.

Often expressed as a percentage of direct costs, direct and indirect costs, direct and indirect costs plus the land value, or the value of the completed project, entrepreneurial profit is market sensitive and varies with changes in economic conditions.

The use of the quantity survey method requires an extensive knowledge of building construction. (See Figure 3.4.)

FIGURE 3.4
Itemized Costs

Direct Costs	
Clearing the land	$ 723
Rough and fine grading	16,753
Footings	8,760
Slabs	8,341
Entrance and stoops	4,200
Balconies	12,350
Interior stairs	575
Dampproofing	938
Rough and finished carpentry	127,390
Furring	3,480
Doors	3,560
Rough and finished hardware	11,722
Kitchen cabinets	14,250
Flooring	18,764
Refrigerators	11,920
Disposals	2,330
Gas ranges	7,386
Venetian blinds	4,800
Bathroom tile	7,297
Painting	13,750
Insulation	1,070
Glazing	2,483
Structural steel and lintels	4,747
Ornamental iron	8,190
Masonry	79,423
Drywall	35,961
Heating and air-conditioning	39,467
Plumbing	44,200
Electrical wiring	21,870
Water mains	4,102
Water and sewer connections	790
Roofing and sheet metal	6,200
Incidentals	9,340
Cleaning, general and contract	5,900
Landscaping	7,450
Fence	1,800
Temporary utilities	4,728
Temporary roads and structures	2,857
Streets, parking area, sidewalks, curbs, and gutters	33,689
Sanitary and storm sewers	15,042
Supervision and time keeping	21,785

Indirect Costs	
Permit	$ 35
Survey	1,980
Layout	465
Payroll taxes and insurance	8,570
Builder's overhead and profit	64,000

LESSON 2 REVIEW QUESTIONS

1. The _____ method estimates construction cost by multiplying the known original cost of a building by a factor.
2. The use of the _____ method requires an extensive knowledge of building construction.
3. Indirect costs are also referred to as _____ costs.
4. The _____ method of calculating the cost improvements is also known as component costing or the segregated cost method.
5. Entrepreneurial profit is also known as _____ profit.
6. With the square-foot method, the structure is broken down into a square-foot _____.
7. The _____ method is the most comprehensive method of costing.
8. _____ costs are also referred to as hard costs.
9. _____ costing is also known as the segregated cost method.
10. With the _____ method, direct costs, indirect costs, and entrepreneurial profit are computed more precisely than in any other method.

LESSON 2 REVIEW ANSWERS

1. cost index
2. quantity survey
3. soft
4. unit-in-place
5. developers'
6. unit of comparison

7. quantity survey
8. Direct
9. Component
10. quantity survey

LESSON 3: Depreciation

Depreciation is a loss in value. While often the term *depreciation* describes all loss of value, the correct term for the loss of value from all causes is **accrued depreciation**. The loss-in-value terms are specific categories of value loss that describe these causes. The definitions are specific, and professional appraisers should take care to use these loss-of-value terms accurately.

PHYSICAL DEPRECIATION

Physical depreciation is the loss in value as a result of general wear and tear. Items that are considered **short-lived items** are those that will wear out before the useful life of the property has passed. *Useful life* is the amount of time that the existing improvements can be reasonably expected to contribute utility under which the property was designed. *Physical life* is the amount of time that the improvements will still be habitable. The *economic life* is the amount of time that the improvements will contribute to the value over the salvage value. Most often the physical life extends beyond the economic life of a property. As an example, the carpeting will likely need replacing several times throughout the useful life of a building.

Long-lived items are items that are not likely to be replaced over the useful life of the property. Such items as framing or floor joists are usually too costly to replace. When the cost of repairing or replacing an item exceeds the value return realized by replacing the item, it is said to be **incurable,** hence the term **incurable physical depreciation.**

Conversely, if the value return exceeds the cost of repairing or replacing the worn-out item, then it is said to be **curable,** hence the term **curable physical depreciation.** Curable physical depreciation is also referred to as *deferred maintenance.* The term *cost-to-cure* is reflective of the costs associated with repairing or replacing a defective item.

OBSOLESCENCE

Another form of value loss is *obsolescence.* **Obsolescence** is the negative market-response to an item. There are two types of obsolescence: *functional obsolescence* and *external obsolescence* (also called *economic obsolescence*). **Functional obsolescence** is a value loss as a result of the market's negative

response to some functionality of the property. This type of obsolescence is a loss in value due to something that exists within the property itself. For example, a poor floor plan or a peculiar carpet color and style that were requested by the market. Suppose that the building is recently constructed. There is no physical depreciation, as both of these items are new, but there is a value loss as the market is not willing to pay the typical price for the building because of the poor floor plan. Likewise, the market is also not willing to pay the typical price for carpeting deemed "peculiar." Because replacing the floor plan would not likely be easy without an excessive *cost-to-cure*, the property suffers from **incurable functional obsolescence.** Suppose that replacing the property's carpet with a more desired floor covering is $2,500 and that the property would receive a value return equaling or exceeding $2,500, then the property has **curable functional obsolescence** of $2,500.

Suppose that the market supports a loss of value of $15,000 from the incurable floor plan. The functional obsolescence calculation is therefore $17,500 ($15,000 + $2,500). Had there been a loss of value as a result of physical depreciation, adding this figure to the obsolescence figures calculates the accrued depreciation. Because there is no physical depreciation, the accrued depreciation remains as $17,500.

Sometimes there may a loss in value as a result of an *overimprovement*. This is a functional obsolescence that results in a **superadequacy.** In such cases the market will not pay the additional costs associated with the item. For example, high-quality gold plumbing fixtures in a tract home would be extravagant items for which the market would not likely pay additional funds.

External obsolescence is a value loss realized as a result of the market's negative response to something that is outside of the property lines. External obsolescence is never curable, as the ability to rectify the issue is beyond the control of the property owner.

METHODS OF DEPRECIATION

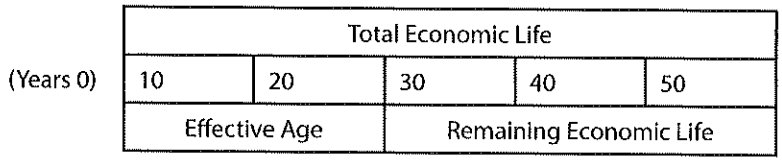
Economic Age-Life Method

Perhaps the single most recognized and used method of estimating depreciation in the appraisal field is the *economic age-life method*. The **economic age-life method** of depreciation (also simply referred to as the *age-life method*) is a method of calculating a percent of depreciation by dividing the *effective age* by the *total economic life*:

$$\frac{\text{Effective age}}{\text{Total economic life}} = \%\text{ of depreciation}$$

Effective age (EA) is the age that the property appears to be as of the date of inspection. **Total economic life (TEL)** is the *effective age* plus the *remaining economic life*. The **remaining economic life (REL)** is the time that the existing improvements are likely *to contribute economically* to the value of the property.

For example, if the effective age of a structure is 20 years and the remaining economic life is 30 years, the total economic life estimate is 50 years as illustrated below.



Therefore, the formula for estimating accrued depreciation by the economic age-life method may also be expressed as:

$$\frac{\text{Effective age}}{\text{(Effective age + Remaining economic life)}} \times \text{Cost new} = \text{Accrued depreciation}$$

Five steps summarize the economic age-life method of estimating accrued depreciation:

1. Estimate the total reproduction or replacement cost new of improvements
2. Estimate the effective age **(EA)** of the property
3. Estimate the total economic life **(TEL)** of the property
4. Estimate the rate of depreciation (depreciation percentage) by dividing effective age by total economic life **(EA ÷ TEL = % of depreciation)**
5. Multiply the rate of depreciation by the total cost new of the improvements to arrive at the estimated accrued depreciated amount

The *depreciated cost of the improvements* can now be computed by subtracting the estimated accrued depreciation amount from the estimated cost new.

For example, suppose a property was constructed 35 years ago. That property has a *chronological age* or *actual age* of 35 years. **Chronological age** (also called **actual age**) is the age that the property has actually been in existence from the time of its original construction. Suppose also that because of proper maintenance and property management the property looks to be effectively 15 years old, and that the estimated time that the existing building improvements will continue to contribute under the current use is another 45 years. Then the effective age (EA) is 15 years, the remaining economic life (REL) is 45 years, and the total economic life (TEL) is estimated at 60 years (15 + 45 = 60). Therefore, the estimate of the percent

of depreciation under the age-life method is 25 percent (15 ÷ 60 = 0.25). If it were estimated that the subject would have a replacement cost new of $100,000, then the depreciation using the economic age-life method would be calculated as $25,000 ($100,000 × 0.25 = $25,000).

Subtracting the estimated accrued depreciation amount from the estimated cost new now computes the depreciated cost of the improvements.

Estimated cost	$100,000
minus depreciation	– 25,000
Depreciated cost of the improvements	$75,000

The benefit of this method is that it is widely accepted, reasonably accurate, and fairly simple to execute. Some of the drawbacks are that it is somewhat subjective, does not isolate a breakdown of the depreciation, and assumes that the building will depreciate at a projected straight-line rate into the future (depreciate the same amount per year).

■ In Practice

The estimated reproduction cost of a commercial building is $600,000. This type of building typically has a total economic of 55 years. The actual age and the effective age are 8 years. Using the economic age-life method, calculate the depreciation percentage, the depreciation amount, and the depreciated cost of the improvement.

Solution:

Using the five steps previously mentioned:

1. Estimate the cost new of improvements: $600,000
2. Estimate the effective age (EA) of the property: 8 years
3. Estimate the total economic life (TEL) of the property: 55 years
4. Estimate the rate of depreciation (depreciation percentage) by dividing effective age by total economic life (EA ÷ TEL = % of depreciation):

   8 ÷ 55 = 0.15 or 15%

5. Multiply the rate of depreciation by the total cost new of the improvements to arrive at the estimated accrued depreciated amount:

   $600,000 × 0.15 = $90,000

The depreciated cost of the improvements:

$600,000 – $90,000 = $510,000

Modified Economic Age-Life Method

The **modified age-life method** of depreciation is similar to the economic age-life method except that the curable items such as curable physical depreciation and curable functional obsolescence are removed before the ratio of effective age to total economic life is applied. Any changes in the effective age are considered a result of curing the physical and functional defects. The result is a more accurate representation of how the market will respond to a property with known curable items that can be isolated and quantified.

For example, suppose that a property has a curable cost-to-cure from both physical depreciation and functional obsolescence of $30,000, and a total estimated cost of $400,000. The actual and effective age of the property is 20 years, but because the repairs will lower the effective age to ten years and the remaining economic life estimate is 45 years, the depreciation percent is 18 percent (EA divided by TEL [10 ÷ 55], where TEL equals EA added to REL [10 + 45 = 55]).

The following is the flowchart of calculations:

Total cost	$400,000
Cost-to-cure	– 30,000
Remaining cost	$370,000
Remaining cost × depreciation percent	– 66,600
Depreciated cost new modified age-life	$303,400

Adding the depreciated cost new of $303,400 to the land value yields an indicated value by the cost approach using the modified economic age-life method of depreciation.

Breakdown Method

The **breakdown method**, also known as the *observed condition method* or the **engineering breakdown method**, is the most detailed and comprehensive method of calculating depreciation. This method estimates each of the five forms of depreciation:

1. Curable physical depreciation, also known as deferred maintenance
2. Incurable physical depreciation

3. Curable functional obsolescence
4. Incurable functional obsolescence
5. External obsolescence

All are individually calculated and then summed to determine the accrued depreciation.

Straight-Line Method

The **straight-line method** of depreciation is the simplest method and is calculated by dividing the cost new (or 100 percent) by the useful life to find an equal annual percentage of depreciation over the life of the asset. As an example, if the estimated cost of a building is $100,000 and the expected useful life is 40 years, then the estimated annual depreciation is $2,500 or 2.5 percent per year ($100,000 ÷ 40 = $2,500 or 2.5%). Straight-line depreciation is not recommended, as most assets do not depreciate in equal amounts year by year. Typically, structures depreciate slowly at first and then more rapidly at the end of the physical life.

Market Method

The **market method,** or *market extraction method,* of determining accrued depreciation finds similar properties and measures their depreciation. Once depreciation as a percent is isolated, this percentage is applied to the subject cost figures to conclude a depreciated cost new of the subject. The main drawback of this method is that the chosen properties from the market must be very similar in quality and condition to the subject. The following steps extract accrued depreciation from the market:

1. Locate a minimum of two improved comparable sales from the market. These sales should appear to suffer from the same amount of depreciation, as does the subject.
2. Deduct the estimated site value from the sales prices of the comparable sales. The result is the contributory value of the improvements only.
3. Estimate the replacement or reproduction cost new (as of the date of sale) for each comparable sale.
4. Deduct the estimated contributory value of the improvements from step 2 from the estimated RCN in step 3. This results in a lump-sum dollar amount of depreciation.
5. Convert the dollar amount arrived at in step 4 into a percentage. Do this by dividing the lump-sum dollar amount of depreciation by the RCN arrived at in step 3.
6. Estimate the effective age of the comparable property.
7. Convert the lump-sum percentage of depreciation into an annual rate of accrued depreciation. Do this by dividing the lump-sum percentage calculated in step 5 by the effective age in step 6.

8. Calculate the mean of the annual rate of accrued depreciation. Apply this average annual rate of accrued depreciation by multiplying by the effective age of the subject property. This results in an indicated percentage of accrued depreciation.
9. Multiply the indicated percentage of accrued depreciation by the RCN of the subject property. This results in a total accrued depreciation dollar amount.

For example, use one comparable sale obtained by the market that sold for $320,000 and has an estimated site value of $95,000. The estimated effective age of this comparable sale is ten years and the estimated RCN is $260,000. Take the following steps to arrive at an annual rate of accrued depreciation:

Step 1	Sales price of comparable	$320,000
Step 2	Minus the estimated site value	− 95,000
	Contributory value of the improvements	$225,000
Step 3	Estimated RCN	$260,000
	Minus contributory value of the improvements	− 225,000
	Lump-sum dollar amount of accrued depreciation	$35,000
Step 4	Lump-sum dollar amount of accrued depreciation	$35,000
	Divided by RCN	÷ $260,000
	Lump-sum accrued depreciation %	0.1346
Step 5	Lump-sum accrued depreciation %	0.1346
	Divided by the effective age in years	÷ 10
Step 6	Average annual rate of accrued depreciation	**0.0135** *or* **1.35% per year**

LAND OR SITE VALUATION TECHNIQUES

The last step before reconciling to an indicated value conclusion under the cost approach is to add the value of the land to the depreciated cost new of the property. Raw land is distinctly different from a site in that a site has

improvements that allow for a certain use. In the cost approach one may either add the cost of land plus the cost of the site improvements, or simply add the cost of the site. Either way is acceptable provided that the reader clearly understands which method was performed, and that all of the costs are accounted for without double counting the site improvements.

As covered in Section 2, there are six methods of determining site value:

1. *Sales comparison method*—Obtain sales of similar vacant sites from the market. Arrive at a final estimate of value for the subject site as though vacant after making any necessary adjustments for differences.
2. *Allocation method*—When a property is improved, apply the ratio of land to building value of similar properties in the market to the subject improved property to arrive at the land value of the subject.
3. *Extraction method* or *abstraction method*—When the sales price and improvement costs of a property are known, subtract the costs from the sales price to arrive at a land value.
4. *Subdivision development method*—Subtract the costs of developing and subdividing a parcel of land from the expected sales prices of the subdivided sites. Make adjustments for time needed to sell all subdivided sites in order to derive the value of the undivided raw land.
5. *Ground rent capitalization method—A ground lease* refers to the agreement between a landowner and a tenant agreeing to erect a building on the site. To derive a land value, capitalize the net income associated with the ground lease. For example, if a ground lease generates an annual income of $50,000 and the market reflects a capitalization rate of 9 percent, the value using the IRV formula is:

$$\text{Value (V)} = \text{Income (I)} \div \text{Capitalization rate (R)}$$
$$= \$50{,}000 \div 0.09$$
$$= \$555{,}555$$

6. *Land residual method*—With this method, the income from the building is deducted from the total net income derived from the entire property (land and building). The balance is the income derived from just the land, which is then capitalized using the IRV formula.

INDICATED VALUE UNDER THE COST APPROACH

Once the land or site value has been added to the depreciated cost new of the improvements, the final step in the cost approach is to conclude an indicated value. The basis for this value indication may or may not be in reality market-driven forces that are present at the time of the assignment. As investigated earlier, there are strengths and weaknesses of the

cost approach, and sometimes the concluded value from the approach may not be what the property is truly worth in the market. For this reason, the often-quoted maxim that "cost does not necessarily equal value" is stated.

The strengths of the cost approach are that it is relatively easy to produce an accurate cost using resources such as cost manuals or proprietary software. Also stated earlier, in the case of a new property it is likely that the cost is closely indicative of the value in the market. Nevertheless, this is not always the true. For this reason, appraisers use more than one approach to value and apply the theory behind economic principles known as *the appraisal principles.*

Some of the weaknesses of the cost approach are that there may be too much depreciation to accurately measure, or that the land value may be extremely difficult to determine, thereby causing the approach to lose validity.

LESSON 3 REVIEW QUESTIONS

1. The modified age-life method of depreciation is similar to the economic age-life method except that the _____ items are removed before the ratio of effective age to total economic life is applied.

2. _____ is the negative market response to an item.

3. Most often the _____ life extends beyond the _____ life of a property.

4. The _____ depreciation method is the simplest method.

5. _____ age is also referred to as actual age.

6. With the _____ method, the income from the building is deducted from the total net income derived from the entire property.

7. Physical depreciation is the loss in value as a result of general _____.

8. _____ is defined as the age that the property appears to be as of the date of inspection.

9. _____ is the effective age plus the remaining economic life.

10. The last step before reconciling to an indicated value conclusion under the cost approach is to add the value of the _____ to the depreciated cost new of the property.

LESSON 3 REVIEW ANSWERS

1. curable

2. Obsolescence

3. physical; economic

4. straight-line

5. Chronological
6. land residual
7. wear and tear
8. Effective age
9. Total economic life
10. land/site

SECTION 3 REVIEW QUESTIONS

1. The principle of substitution as it applies to the cost approach states that
   a. the sales price of the subject must equal the cost.
   b. no person is justified to pay more than the cost new.
   c. the listing price of the subject should be less than the cost.
   d. the asking price should be roughly equal to the cost.

2. Which of the following is a category of depreciation?
   a. Functional depreciation
   b. Economic or external depreciation
   c. Functional obsolescence
   d. Physical obsolescence

3. When there is a loss in value due to a poor layout, the property is said to suffer from
   a. physical depreciation.
   b. functional obsolescence.
   c. functional depreciation.
   d. layout depreciation.

4. A loss in value as a result of the market's negative response to something that is outside of the property lines is
   a. functional obsolescence.
   b. economic depreciation.
   c. physical depreciation.
   d. external obsolescence.

5. The cost approach includes
   a. the value of land as though vacant.
   b. an estimate of cost new.
   c. an estimate of accrued depreciation.
   d. All of the above

6. Functional obsolescence that is a result of an overimprovement is referred to as a(n)
   a. hyperadequacy.
   b. overadequacy.
   c. superadequacy.
   d. None of the above

7. The formula to calculate annual depreciation using the straight-line method is
   a. expected life divided by cost new.
   b. cost new divided by expected life.
   c. effective age divided by total economic life.
   d. None of the above

8. If the cost new is $200,000 and the estimated expected life is 50 years, what is the annual depreciation using the straight-line method?
   a. $3,000
   b. $2,000
   c. $4,000
   d. $25,000

9. The formula for the economic age-life method of depreciation is
   a. effective age divided by remaining economic life.
   b. effective age divided by effective age plus remaining economic life.
   c. effective age divided by total economic life.
   d. Both b and c

10. A home that was built 36 years ago and has an effective age of 30 years is said to have a chronological age of
    a. 30 years.
    b. 36 years.
    c. 66 years.
    d. None of the above

11. Total economic life is
    a. remaining economic life minus effective age.
    b. actual age minus effective age.
    c. effective age plus remaining economic life.
    d. actual age minus remaining economic life.

12. The most detailed and comprehensive method of calculating depreciation is the
    a. age-life method.
    b. breakdown method or engineering method.
    c. unit-in-place method.
    d. straight-line method.

13. The most detailed method of estimating costs is the
    a. unit-in-place method.
    b. economic age-life method.
    c. quantity survey method.
    d. square-foot method.

14 A loss in value from all causes of depreciation is known as
    a. physical depreciation.
    b. functional obsolescence.
    c. external obsolescence.
    d. accrued depreciation.

15. A commercial building that was built 13 years ago and has an effective age of 8 years is said to have an actual age of
    a. 10 years.
    b. 21 years.
    c. 8 years.
    d. None of the above

section four

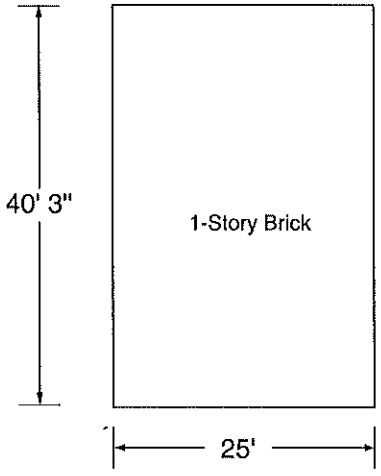
IV

RESIDENTIAL CASE STUDIES

CASE STUDY #1

Your client has asked you to appraise a two-bedroom single-family home located at 1405 S. 51st Court, Hometown, USA. The purpose of the appraisal is mortgage funding and you are to use all three approaches to value including the cost approach. The subject, built five years ago, has a living room, kitchen, dining room, one bathroom, two large bedrooms, and a 22'× 24' detached brick garage, all situated on a 30' × 125' lot. This one-story brick home does not have a basement.

The improvements have an estimated total economic life of 65 years. The actual age of the improvements is 5 years and the quality of construction is good. After visiting the subject, you estimate that the subject has a remaining economic life of 60 years. You have confirmed all dimensions and the sketch of the subject follows:



Land Valuation

There are no comparable vacant land sales within the subject's neighborhood; however, there are comparables sales of improved properties similar to the subject. Accrued depreciation for improved properties similar to the subject in age, size, condition, and appeal is an estimated 10 percent. The estimated cost of improvements for all four comparables using the cost approach is in the following table:

Sale	Sales Price	Cost of Improvements (RCN)	Depreciated Cost of Improvements	Land Value
1	$210,000	$196,667		
2	$216,000	$200,000		
3	$209,000	$189,444		
4	$220,000	$203,889		

1. What is the estimated value of the subject site as though vacant using the extraction method?

Replacement/ Reproduction Cost New

The subject's interior walls are 8 feet high and the exterior walls are 8.5 feet high. The estimated cost of site improvements is $3,500. Using the following information, calculate the cost to construct using the unit-in-place method. (Note: round GLA to the nearest one.)

Foundation:	concrete walls and footing @ $65 per linear foot
Floor framing:	$7.50 per sq. ft. of floor area
Roof:	$8.25 per sq. ft. of roof area
Interior finishes:	ceiling, partition walls, interior painting $16,200;
	floor covering $6,600;
	cabinets, fixtures $36,500
Front and rear exterior walls:	face brick on concrete block @ $28.85 per sq. ft. of wall area (25' × 8.5');
	one entrance door 3' × 6'8" @ $400;
	one 4' × 6' window @ $350

Side exterior walls:	face brick on concrete block @ $28.85 per sq. ft. of wall area (40.25' × 8.5');
	windows on 20% of wall area @ $20.25 per sq. ft.
Wall framing:	$72.32 per linear foot of building perimeter
Electrical:	$29.45 per sq. ft. of GLA
Heating/cooling:	$12,600
Plumbing:	$17.55 per sq. ft. of GLA
Garage:	$32 per sq. ft.

2. What is the replacement cost new of the improvements (home and garage)?

Depreciation

The floor plan is typical for this type of property in this market and you observe that that are no obvious superadequacies; therefore, your subject does not suffer from any functional obsolescence. No external obsolescence is observed.

3. Using the effective age life method, calculate the percentage of physical depreciation.
4. What is the total physical depreciation amount?
5. What is the indicated value of the subject using the cost approach?

Solution

1. The typical land/site value = $36,000

Sale	Sales Price	Cost of Improvements (RCN)	Depreciated Cost of Improvements	Land Value
1	$210,000	$196,667	$177,000	$33,000
2	$216,000	$200,000	$180,000	$36,000
3	$209,000	$189,444	$170,500	$38,500
4	$220,000	$203,889	$283,500	$36,500
Mean				$36,000

*Calculate the depreciated cost of the improvements by deducting the estimated accrued depreciation percentage (10%) from the RCN. Calculate land value by subtracting depreciated cost of improvements from the sales price.

2. RCN using unit-in-place costs:

Foundation:		
Perimeter (2 × 25) + (2 × 40.25) = 130.5 ft.		
130.5 ft. @ $65.00		$8,482.50
Floor framing:		
1,006 sq. ft. @ $7.50		$7,545.00
Roof:		
1,006 sq. ft. @ $8.25		$8,299.50
Interior finishes:		
Ceiling, partition walls, and interior painting:		$16,200.00
Floor covering:		$6,600.00
Cabinets and fixtures:		$36,500.00
Front and rear exterior walls:		
25′ × 8.5′ = 212.5 sq. ft. × 2 = 425 sq. ft.		
– 40 sq. ft. for doors (20 sq. ft. × 2)		
– 48 sq. ft. for windows (24 sq. ft. × 2)		
337 sq. ft. @ $28.85	$9,722.45	
2 doors @ $400	$800.00	
2 windows @ $350	$700.00	$11,222.45
Side exterior walls:		
2 (40.25′ × 8.5′) = 684 sq. ft. – 137 sq. ft. (20% for windows)		
= 547 sq. ft. @ $28.85	$15,792.49	
137 sq. ft. @ $20.25	$2,774.25	$18,566.74
Wall framing:		
Perimeter (2 × 25) + (2 × 40.25) = 130.5 ft.		
130.50 ft. @ $72.32		$9,437.76
Electrical:		
1,006 sq. ft. @ $29.45		$29,626.70
Heating/cooling:		$12,600.00
Plumbing:		
1,006 sq. ft. @ $17.55		$17,655.30
Total		**$182,735.95**

Total replacement/reproduction cost (RCN):	
Home:	**$182,735.95**
Garage:	
528 sq. ft. @ $32	**$16,896.00**
Total	**$199,631.95**

3. Because the estimated total economic life of the improvements is 65 years, and there is an estimated remaining economic life of 60 years, the effective age is five years (65 – 60).

$$\frac{\text{Effective age}}{\text{Total economic life}} = \% \text{ of depreciation}$$

$$= 5 \div 65 = 0.08 \text{ or } 8\%$$

4. Physical depreciation = RCN × % of depreciation

$$= \$199{,}631.95 \times 0.08 = \$15{,}970.56$$

5.

$199,632	Replacement cost new (RCN)
– 15,971	Physical depreciation
– 0	Functional obsolescence
– 0	External obsolescence
$183,661	Depreciated cost of improvements
+ 36,000	Site value
+ 3,500	Site improvements
$223,161	Indicated value using the cost approach

CASE STUDY #2

The subject is a 2,400-square-foot one-story single-family property with a 24' × 24' garage. The house is situated on a one-acre site with additional site improvements worth $5,500. The building is 25 years old with an effective age of 15 years and a remaining economic life of 55 years. The subject has two fireplaces that cost $5,000 each to build at today's replacement cost. The subject is considered to be good quality construction. The following costs to build the property are taken from a leading cost guide:

Good Quality

2,000 sq. ft. = $81.50/sq. ft.

2,500 sq. ft. = $80.00/sq. ft.

appendix

APPRAISAL MATH AND STATISTICS

LESSON 1: Basic Mathematical Functions

LESSON 2: Calculating Area

LESSON 3: Statistics Overview

LESSON 4: Mathematics of Finance

LEARNING OBJECTIVES

By the end of this lesson, participants will be able to

- explain the order of operations;
- add, subtract, multiply, and divide negative numbers;
- calculate percentages and convert into decimals;
- understand reciprocals;
- round numbers;
- understand exponents;
- understand problem-solving techniques;
- comprehend basic algebra concepts;
- understand interpolation and extrapolation;
- understand units of comparison;
- calculate area and volume;
- convert various units of measure;
- understand the difference between population and sample;
- understand measures of central tendency;

- calculate the mean, median, and mode of an array of numbers;
- understand measures of variability;
- calculate range, variance, and standard deviation;
- calculate weighted averages;
- understand the financial principle of time value of money;
- distinguish between simple interest and compound interest;
- understand each of the six functions of a dollar;
- use the various methods to compute each function of a dollar; and
- calculate a mortgage payment.

KEY TERMS

above grade area
additive inverse
algebraic expression
annuity
area
array
compounding
compounding period
cubic units
data set
decomposition
depth
digit
discounting
discount rate
division rule
Ellwood Table
exponent
extrapolation
frontage
front foot
front footage
future value
future value of $1
future value of $1 per period
gross living area
interpolation
least common denominator (LCD)
linear measurement
mean
measures of central tendency
measures of variability
median
mixed-sign rule
mode
multiplication rule
multiplicative inverse
negative number
nominal data
order of operations
outliers
parameter
payment to amortize $1
percent/percentage
perimeter
place value
population
positive number
present value
present value of $1
present value of $1 per period
prism
qualitative
quantitative
range
reciprocal
regression analysis
reversion
rounding
rounding digit
rounding off
same-sign rule
sample
sinking fund factor
six functions of a dollar
square
square root
standard deviation
statistic
statistics
time value of money
unit of comparison
variance
volume
weighted average
width

SUGGESTED READING

Fundamentals of Real Estate Appraisal, 9th Edition, William L. Ventolo Jr. and Martha R. Williams, Dearborn Real Estate Education (2005), Chapter 15

Basic Appraisal Principles, Mark A. Munizzo and Lisa Virruso Musial, Dearborn Real Estate Education (2007)

- Calculate the next innermost enclosure, remembering the order of operations, and then plug the results into the equation:

$$7 \times [333]$$

- Calculate the final outermost enclosure, and then plug the results into the equation and solve the equation:

$$7 \times 333 = 2{,}331$$

For example: Calculate multiplication and division from left to right, whichever comes first.

Correct	Incorrect
$6 \times 9 \div 27$	$6 \times 9 \div 27$
$(6 \times 9) \div 27$	$6 \times (9 \div 27)$
$= 54 \div 27$	$= 6 \times 0.33$
$= 2$	$= 1.98$

WHAT ARE NEGATIVE AND POSITIVE NUMBERS?

Negative numbers are numbers that have a minus sign (-) in front of them, meaning that the number is less than zero. **Positive numbers** are numbers that have a plus sign (+) in front of them, meaning that the number is greater than zero. Usually, a positive number is expressed without a sign in front of it. In other words +16 is the same as 16.

To simplify, suppose you have $1,000 in a checking account. Your bank balance would read +$1,000. Now you decide to withdraw $1,000. Your bank balance now reads $0. You then write a check for $1,000 causing an overdraft of $1,000. Your balance now reads -$1,000; in other words, $1,000 less than zero.

To perform mathematical calculations with negative and positive numbers, you must understand the same-sign rule, the mixed-sign rule, subtracting and adding negative numbers, and multiplying and dividing negative numbers.

Same-Sign Rule

The **same-sign rule** is: When combining numbers with the same sign, keep the sign and then add.

$$(+a) + (+b) = +(a + b)$$

$$(-a) + (-b) = -(a + b)$$

To simplify, take the following steps:

1. First, determine if the numbers are positive or negative.

   $$(+a) + (+b) = ? \text{ or } (-a) + (-b) = ?$$

2. Place the sign of the numbers in front of a set of parentheses.

   $$+(\quad) \text{ or } -(\quad)$$

3. Write the numbers inside the parentheses and place a plus sign (+) between them.

   $$+(a + b) \text{ or } -(a + b)$$

4. Add the numbers and remove the parentheses.

■ In Practice

Solve the following:

+11 + 17 = ?

Solution:

+11 + 17 = ?

= +(11 + 17)

= +28

■ In Practice

Solve the following:

-11 – 17 = ?

Solution:

-11 – 17 = ?

= -(11 + 17)

= -28

■ **In Practice**

Solve the following:

-3 – 10 – 6

Solution:

-3 – 10 – 6

= -(3 + 10 + 6)

= -19

Mixed-Sign Rule

The **mixed-sign rule** is not quite as simple as the same-sign rule. When combining two numbers with different signs, take the following steps:

+a – b = ? or -a + b = ?

1. First, ignoring the signs, determine which of the two numbers is larger and place the sign of that number in front of a set of parentheses.

   +( ) or -( )

2. Then inside the parentheses, subtract the smaller number from the larger number.

■ **In Practice**

Solve the following:

+10 – 2 = ?

Solution:

- First, ignoring the signs, determine which of the two numbers is larger and place the sign of that number in front of a set of parentheses.

  +( )

- Then inside the parentheses, subtract the smaller number from the larger number.

  = +(10 – 2)

  = +(8)

  = +8

In Practice

Solve the following:

$$-10 + 2 = ?$$

Solution:

1. First, ignoring the signs, determine which number is larger and place the sign of that number in front of a set of parentheses.

$$-( )$$

2. Then inside the parentheses, subtract the smaller number from the larger number.

$$= -(10 - 2)$$

$$= -(8)$$

$$= -8$$

In Practice

Solve the following combining the same-sign rule and the mixed-sign rule.

$$-3 + 8 - 8 + 11 + 6 - 9 = ?$$

Solution:

1. First, group the positive and negative numbers separately.

$$+8 + 11 + 6 \qquad -3 - 8 - 9$$

2. Next, apply the same-sign rule on each group, by placing the sign of each group number in front of a set of parentheses, writing the numbers inside the parentheses, and placing a plus sign (+) between them.

$$+(8 + 11 + 6) - (3 + 8 + 9)$$

3. Next, apply the mixed-sign rule by determining the sign of the larger number, placing it outside parenthesis, and subtracting the smaller number from the larger number.

$$+ (25) - (20)$$

$$= +(25 - 20)$$

$$= +5$$

ADDING AND SUBTRACTING NEGATIVE NUMBERS

Adding a Negative Number

Some equations have two signs next to each other with no numbers between them.

$$5 + (-9)$$

To add a positive and a negative number:

1. First, ignore the signs and find the positive difference between the numbers.
2. Then attach the sign of the original larger number.

■ **In Practice**

Solve the following:

5 + (-9) = ?

Solution:

1. First, ignore the negative sign and find the positive difference between 5 and 9.

   9 – 5 = 4

2. Then attach the sign of the original larger number (9), which is a minus sign (–). Therefore:

   5 + (-9) = -4

Subtracting a Negative Number

All negative numbers have a positive counterpart known as its additive inverse. An **additive inverse** is a number that when added to another number equals zero. For example, the additive inverse of -10 is +10. Adding -10 to its additive inverse (+10) results in zero. Therefore, adding a negative number is the same as subtracting its positive counterpart (additive inverse) and subtracting a negative number is the same as adding its positive counterpart (additive inverse).

■ **In Practice**

Solve the following:

8 – (-3) = ?

Solution:

$$8 - (-3) = ?$$

$$= 8 + (+3) = 11$$

MULTIPLYING AND DIVIDING NEGATIVE NUMBERS

Multiplication Rule

The **multiplication rule** states that when multiplying two numbers with the same sign, the result is positive, and when multiplying two numbers with different signs, the result is negative.

For example:

$$8 \times (-6) = -48$$

$$2 \times 2 = 4$$

$$-12 \times (-2) = 24$$

Division Rule

The **division rule** states that when dividing two numbers with the same sign, the result is positive, and when dividing two numbers with different signs, the result is negative.

For example:

$$45 \div (-3) = -15$$

$$100 \div 20 = 5$$

$$-30 \div -5 = 6$$

■ **In Practice**

Solve the following equation using the multiplication and division rules, remembering the order of operations.

$$12 \times (-3) \div 2 = ?$$

Solution:

$$12 \times (-3) \div 2 = ?$$

$$(-36) \div 2 = -18$$

PERCENTAGES AND DECIMALS

Denoted by a percent sign (%), **percent** means divided by 100, per 100, or per 100 parts (100 parts = 1 whole). For example, 25% means 25 ÷ 100 or ¼, which translates into 0.25.

- To convert a percentage to a decimal, just divide by 100, or just move the decimal point two places to the left, adding zeros if needed, and remove the percent sign (%).

For example, to convert 60% to a decimal, move the decimal point to the left two spaces, and drop the percentage sign (%). The result is 0.60.

- To convert a decimal to a percentage, multiply by 100 or just move the decimal point two spaces to the right and add a percentage sign.

For example, to convert 0.0875 to a percentage, multiply 0.0875 by 100, or move the decimal point to the right two spaces, and add a percentage sign (%). The result is 8.75%.

In Practice

Convert the following percentages to decimals:

1. 74%
2. 14%
3. 16.5%
4. 0.01%
5. 8¾%

Solution:

Simply move the decimal point two spaces to the left and remove the percent sign, or divide by 100.

1. 74% = 74 ÷ 100 = 0.74
2. 14% = 14 ÷ 100 = 0.14
3. 16.5% = 16.5 ÷ 100 = 0.165
4. 0.01% = 0.01 ÷ 100 = 0.0001
5. 8¾% = 8 + (3/4)% = 8 + (3 ÷ 4)% = 8 + 0.75% = 8.75% = 8.75 ÷ 100 = 0.0875

■ **In Practice**

Convert the following decimals to percentages.

1. 0.01
2. 0.99
3. 0.0066
4. 0.50
5. 0.0001

Solution:

Simply move the decimal point two spaces to the right and add a percent sign, or multiply by 100.

1. $0.01 = 0.01 \times 100 = 1\%$
2. $0.99 = 0.99 \times 100 = 99\%$
3. $0.0066 = 0.0066 \times 100 = 0.66\%$
4. $0.50 = 0.50 \times 100 = 50\%$
5. $0.0001 = 0.0001 \times 100 = 0.01\%$

PERCENTAGE PROBLEMS

To find a percentage of a number, just convert to a decimal, and multiply.

■ **In Practice**

What is 50 percent of 700?

$$\mathbf{50\% \times 700 = ?}$$

Solution:

First convert the percentage to a decimal, and then multiply by 700.

$$\mathbf{50\% \times 700 = ?}$$

$$\mathbf{(50 \div 100) \times 700 = 0.50 \times 700 = 350}$$

There are three variables to solve percentage problems:

- Percent (rate)
- Total
- Part

Using the preceding problem:

Percent (rate) = 50%

Total = 700

Part = 350

State the problem as follows:

Part = Percent × Total

Because there are three variables in this problem, at least two variables must be known. The Percent and the Total represent the known (given) variables in the preceding example. We were solving for the unknown variable (Part).

As with all mathematical formulas, the unknown variable will vary; therefore, we can express the previous formula three different ways depending on which of the three variables are known (given).

Part = Percent × Total $\qquad$ **350** = 0.50 × 700

or

$$\textbf{Percent} = \frac{\text{Part}}{\text{Total}} \qquad \textbf{0.50} = \frac{350}{700}$$

or

$$\textbf{Total} = \frac{\text{Part}}{\text{Percent}} \qquad \textbf{700} = \frac{350}{0.50}$$

It is now obvious that the mixture of unknown variables and known variables determines which equation is used to solve the problem.

■ In Practice

$60,000 represents what percent of $100,000?

Solution:

$$\text{Percent (rate)} = ?$$

$$\text{Total} = \$100{,}000$$

$$\text{Part} = \$60{,}000$$

$$\text{Percent} = \frac{\text{Part}}{\text{Total}} = \frac{\$60{,}000}{\$100{,}000}$$
$$= 0.60 = 0.60 \times 100 = 60\%$$

In Practice

120 represents 50 percent of what number?

Solution:

$$\text{Percent (rate)} = 50\% \text{ or } 0.50$$

$$\text{Total} = ?$$

$$\text{Part} = 120$$

$$\text{Total} = \frac{\text{Part}}{\text{Percent}} = \frac{120}{0.50} = 240$$

In Practice

If you paid $90 for an item that was discounted by 10%, what was the original price?

Solution:

Because $90 is the result after an item is discounted by 10%, it represents 90% (100% – 10%) of the original price. Remember 90% converted into a decimal is expressed as 0.90.

$$\text{Total} = \frac{\text{Part}}{\text{Percent}}$$

$$\text{Total} = \frac{\$90}{0.90} = \$100$$

Therefore the original price was $100.

In Practice

If a recent 8% price increase resulted in a sales price of $750,000, what was the price before the recent increase?

Solution:

- First recognize that $750,000 represents some number that was increased by 8%. In other words, it is the result of a number being increased by 108%.

$750,000 = price before increase × (1.08)

Percent (rate) = 108% or 1.08

Total = price before increase

Part = $750,000

$$\text{Total} = \frac{\text{Part}}{\text{Percent}} = \text{Price before increase} = \frac{\$750{,}000}{1.08}$$

$$= \$694{,}444$$

Therefore, the price before the increase was $694,444.

PERCENTAGE INCREASES AND DECREASES

Always calculate percentage increases and decreases with respect to the value before the change took place. For example, suppose a home sold for $450,000 in January of 2006 and resold in January of 2008 for $560,000. What is the percentage increase in the sales price?

1. Calculate the actual difference (the increase or decrease) in the sales price from January of 2006 to January of 2008.

$$\$560{,}000 - \$450{,}000 = \$110{,}000$$

2. Then calculate the percentage increase with respect to the original sales price.

$$(\$110{,}000 \div \$450{,}000 = 0.244) \times 100 = 24.4\%$$

In appraisal problems, use the actual percentage increase or decrease calculation when determining a market condition (time) adjustment in the sales comparison grid (see Lesson 4 of this section).

It could be determined from the previous example, that the property value increased approximately 12.2 percent per year. The actual distribution of the 24.4 percent increase throughout the 24-month period is difficult to determine. In other words, the majority of the increase in value could have taken place within in the last six months prior to January of 2008, or perhaps a 1 percent escalation per month took place.

Place Values

A **digit** is a symbol that is used in the system of numeration (system of numbers). A digit can be one of ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

For example, the number 345 has three digits: 3, 4, and 5.

Each digit of a number has a different **place value**, the name of a place or location of a digit. Place value is determined by its position as a number. Determine the value of a digit by its position in a number.

For example, each digit in the number 345 has a different place value.

345

ones place value (5 ones)

tens place value (4 tens)

hundreds place value (3 hundreds)

Larger numbers have more place values. For example, the number 562,345 has six digits and therefore six place values.

562,345

1s: ones place value (5 ones)

10s: tens place value (4 tens)

100s: hundreds place value (3 hundreds)

1,000s: thousands place value (2 thousands)

10,000s: ten-thousands place value (6 ten-thousands)

100,000s: hundred-thousands place value (5 hundred-thousands)

Decimal numbers have place values as well. The first digit after the decimal point is the tenths (1/10s) place value; the next digit to the right is the hundredths (1/100s) place value; the next digit to the right is the thousandths (1/1,000s) place value; the next digit is the ten-thousandths (1/10,000s) place value; and so on.

For example, each digit after the decimal point in the number 0.6785, has a different place value.

0.6785

1/10,000s: ten-thousandths place value (5 ten-thousandths)

1/1,000s: thousandths place value (8 thousandths)

1/100s: hundredths place value (7 hundredths)

1/10s: tenths place value (6 tenths)

ROUNDING

The process of **rounding off** expresses a rounded number as an expression of an exact number. **Rounding** makes numbers easier to work with and is used when numbers do not have to be exact. It is a form of estimating. To round a number, you must first determine the **rounding digit,** which is the digit that occupies the place value you are rounding to.

Rounding Off Whole Numbers

- **Rule 1**—Determine the rounding digit and look to the right of it. If the digit is less than 5 (0, 1, 2, 3, or 4), do not change the rounding digit. Change all the digits to the right of the rounding digit to 0.
- **Rule 2**—Determine the rounding digit and look to the right of it. If the digit is 5 or greater (5, 6, 7, 8, or 9), round the rounding digit up by one number. Change all the digits to the right of the rounding digit to 0.

Rounding Decimal Numbers

- **Rule 1**—Determine the rounding digit and look to the right of it. If the digit is less than 5, just drop all the digits to the right of it.
- **Rule 2**—Determine the rounding digit and look to the right of it. If the digit is 5 or greater, round the rounding digit up by one number and drop all digits to the right of it.

Rounding Numbers to the Nearest 10

- Round numbers that end in 1 through 4 down to the next lower number that ends in 0. For example, 93 rounded to the nearest ten is 90.
- Round numbers that end in 5 or more up to the next higher even ten. For example, 67 rounded to the nearest ten is 70.

Rounding Numbers to the Nearest 100

- Round numbers that end in 1 through 49 down to the next lower number that ends in 00. For example, 523 rounded to the nearest hundred is 500.
- Round numbers that end in 50 or more up to the next higher even hundred. For example, 989 rounded to the nearest hundred is 1,000.

Rounding Numbers to the Nearest 1,000

- Round numbers that end in 001 through 499 down to the next lower number that ends in 000. For example, 4,433 rounded to the nearest thousand is 4,000.
- Round numbers that end in 500 or more up to the next even thousand. For example, 7,989 rounded to the nearest thousand is 8,000.

Rounding Numbers to the Nearest 10,000

- Round numbers that end in 0001 through 4,999 down to the next lower number that ends in 0000. For example, 64,328 rounded to the nearest ten-thousand is 60,000.
- Round numbers that end in 5,000 or more up to the next even ten-thousand. For example, 87,689 rounded to the nearest ten-thousand is 90,000.

Rounding Numbers to the Nearest 100,000

- Round numbers that end in 00001 through 49,999 down to the next lower number that ends in 00000. For example, 5,447,867 rounded to the nearest hundred-thousand is 5,400,000.
- Round numbers that end in 50,000 or more up to the next even hundred-thousand. For example, 5,555,999 rounded to the nearest hundred-thousand is 5,600,000.

Rounding Decimals to the Nearest Hundredth (0.01)

- If the thousandths place of the decimal is 4 or less, drop it and leave the hundredths place as it is; do not change it. For example, 0.742 rounded to the nearest hundredth is 0.74.
- If the thousandths place is 5 through 9, the hundredths place is increased by 1. For example, 0.637 rounded to the nearest hundredth is 0.64.

Rounding Decimals to the Nearest Tenth (0.1)

- If the hundredths and thousandths places are 49 or less, drop them and leave the tenths place as it is; do not change it. For example, 0.742 rounded to the nearest tenth is 0.7.
- If the hundredths and thousandths places are 50 or more, increase the tenths place by 1. For example, 0.867 rounded to the nearest tenth is 0.9.

For example, 845.2791 becomes:

1,000 when rounding to the nearest thousand (1,000)

800 when rounding to the nearest hundred (100)

850 when rounding to the nearest ten (10)

845 when rounding to the nearest one (1)

845.3 when rounding to the nearest tenth (0.10)

845.28 when rounding to the nearest hundredth (0.01)

845.279 when rounding to the nearest thousandth (0.001)

In Practice

669.3459 becomes:

_____ when rounding to the nearest thousand (1,000)

_____ when rounding to the nearest hundred (100)

_____ when rounding to the nearest ten (10)

_____ when rounding to the nearest one (1)

_____ when rounding to the nearest tenth (0.10)

_____ when rounding to the nearest hundredth (0.01)

_____ when rounding to the nearest thousandth (0.001)

Solution:

669.3459 becomes:

1,000 when rounding to the nearest thousand (1,000)

700 when rounding to the nearest hundred (100)

670 when rounding to the nearest ten (10)

669 when rounding to the nearest one (1)

669.3 when rounding to the nearest tenth (0.10)

669.35 when rounding to the nearest hundredth (0.01)

669.346 when rounding to the nearest thousandth (0.001)

EXPONENTS

An **exponent** indicates how many times a base number is to be used as a factor. For example, 8^3 means 8 raised to the third power, or 8 cubed.

$$8^3 = 8 \times 8 \times 8 = 512$$

The base number is 8 and the exponent is 3. The base can be a whole number, a decimal number, or a fraction. Some rules for using exponents:

- **Rule 1**—A base number that is raised to the power of 1 equals the base number itself.

$$n^1 = n$$

- **Rule 2**—With the exception of 0, any base number raised to the power of zero equals 1.

$$n^0 = 1$$

- **Rule 3**—When multiplying two powers with the same base, simply add the exponents.

$$a^3 \times a^2 = a^{3+2}$$

In Practice

Solve the following:

1. $7^3 = ?$
2. $5^5 = ?$
3. $6^3 = ?$
4. $(0.30)^4 = ?$
5. $10^1 = ?$
6. $3^3 \times 3^2 = ?$
7. $23^0 = ?$
8. $(1/2)^3 = ?$

Solution:

1. $7^3 = 7 \times 7 \times 7 = 343$
2. $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3{,}125$
3. $6^3 = 6 \times 6 \times 6 = 216$
4. $(0.30)^4 = 0.30 \times 0.30 \times 0.30 \times 0.30 = 0.0081$
5. $10^1 = 10$
6. $3^3 \times 3^2 = 3^{3+2} = 3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$
7. $23^0 = 1$
8. $(1/2)^3 = (0.50)^3 = (0.50) \times (0.50) \times (0.50) = 0.125$

SQUARES AND SQUARE ROOTS

The **square** of a number, n, is that number multiplied by itself.

$$n^2$$

For example,

square of $5 = 5 \times 5 = 5^2 = 25$

square of $2 = 2 \times 2 = 2^2 = 4$

square of $4 = 4 \times 4 = 4^2 = 16$

square of $3 = 3 \times 3 = 3^2 = 9$

The **square root** of a number, n, is the number that gives n when multiplied by itself.

$$\sqrt{n}$$

For example,

square root of 25 = 5, because $5 \times 5 = 25$

square root of 4 = 2, because $2 \times 2 = 4$

square root of 16 = 4, because $4 \times 4 = 16$

square root of 9 = 3, because $3 \times 3 = 9$

FRACTIONS

Reciprocals

All numbers, except 0, have a **reciprocal** or **multiplicative inverse**, which is a number that when multiplied by n results in 1.

$$\text{Reciprocal of } n = \frac{1}{n}$$

$$\textit{Proof: } n \times \frac{1}{n} = \frac{(\cancel{n} \times 1)}{\cancel{n}} = 1$$

For example, the reciprocal of $8 = 1/8$

$$\textit{Proof: } 8 \times \frac{1}{8} = \frac{(\cancel{8} \times 1)}{\cancel{8}} = 1$$

The reciprocal of a fraction is the fraction inverted.

$$\text{Reciprocal of } a/b = b/a$$

$$\textit{Proof: } a/b \times b/a = \frac{a \times b}{b \times a} = 1$$

For example, the reciprocal of $3/4 = 4/3$.

$$\textit{Proof: } 3/4 \times 4/3 = \frac{3}{4} \times \frac{4}{3} = \frac{3 \times 4}{4 \times 3} = \frac{12}{12} = 1$$

■ **In Practice**

Find the reciprocal and show proof of the following:

1. 56
2. 6/7
3. 0.90
4. 1
5. 1/0.24

Solution:

1. 1/56; Proof: $56 \times 1/56 = 56/56 = 1$
2. 7/6; Proof: $6/7 \times 7/6 = 42/42 = 1$
3. 1/0.90; Proof: $0.90 \times 1/0.90 = 0.90/0.90 = 1$
4. 1; Proof: $1 \times 1/1 = 1$
5. 0.24; Proof: $1/0.24 \times 0.24 = 0.24/0.24 = 1$

Adding and Subtracting Fractions

All fractions have a *numerator* and a *denominator*. The numerator is the number above the line in a fraction or the top number in a fraction. The denominator is the number below the line in a fraction or the bottom number in a fraction. To add or subtract fractions, all denominators must be the same. This is accomplished by finding the *least common denominator (LCD)* also referred to as the *least common multiple (LCM)*. The **least common denominator** of two or more denominators, other than zero, is the smallest whole number that is divisible by each denominator.

To find the least common denominator:

- First list all multiples of each denominator.
- Then, look for the lowest number that is the same in all lists of denominators—the least (lowest) common denominator.
- Use that number as the denominator for all of the fractions.
- Finally, determine the new numerator for each fraction by first dividing the least common denominator (LCD) by the fraction's denominator and then multiplying that number by the numerator.

$$\text{New numerator} = \frac{\text{LCD}}{\text{Denominator}} \times \text{Numerator}$$

$$\text{New denominator} = \text{LCD}$$

For example:

$$\frac{3}{2} + \frac{5}{4} + \frac{3}{10} = ?$$

- First list all multiples of each denominator:

  Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22...

  Multiples of 4: 4, 8, 12, 16, 20, 24, 28...

  Multiples of 10: 10, 20, 30, 40, 50....

- Then, look for the lowest number that is listed for each denominator.

  Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, **20**, 22...

  Multiples of 4: 4, 8, 12, 16, **20**, 24, 28...

  Multiples of 10: 10, **20**, 30, 40, 50....

  Because 20 is the lowest number that appears on all three lists, it is the least common denominator.

- Finally, find the equivalents of each fraction using the least common denominator, which in this case is 20.

$$\frac{3}{2} + \frac{5}{4} + \frac{3}{10} = ?$$

$$\text{New numerator} = \frac{\text{LCD}}{\text{Denominator}} \times \text{Numerator}$$

New denominator = LCD

new numerator for $\frac{3}{2} = (20 \div 2) \times 3 = \mathbf{30}$

new denominator = 20

Therefore, the equivalent fraction for $\frac{3}{2}$ is $\frac{30}{20}$

new numerator for $\frac{5}{4} = (20 \div 4) \times 5 = \mathbf{25}$

Therefore, the equivalent fraction for $\frac{5}{4}$ is $\frac{25}{20}$

new numerator for $\frac{3}{10} = (20 \div 10) \times 3 = \mathbf{6}$

Therefore, the equivalent fraction for $\frac{3}{10}$ is $\frac{6}{20}$

Now it is possible to calculate the sum of all three fractions because they all have the same denominator.

$$\frac{30}{20} + \frac{25}{20} + \frac{6}{20} = \frac{\mathbf{61}}{\mathbf{20}}$$

Therefore, to add and subtract fractions, first find the lowest common denominator. Convert each fraction to its equivalent value expressed using a common denominator.

Multiplying and Dividing Fractions

To multiply fractions, simply multiply the numerators, and multiply the denominators.

For example:

$$\frac{4}{5} \times \frac{9}{10} = \frac{4 \times 9}{5 \times 10} = \frac{36}{50}$$

To divide fractions, multiply the first fraction (the *dividend*) by the reciprocal of the second fraction (the *divisor*).

For example:

$$(6/8) \div (3/5) = \frac{6}{8} \times \frac{5}{3} = \frac{30}{24}$$

■ **In Practice**

Solve the following:

1. (7/8) – (3/4) = ?
2. (6/9) ÷ (4/5) = ?

Solution:

1. (7/8) – (3/4) = ?

   The LCD = 8

   New numerator for (7/8) = (8 ÷ 8) × 7 = 7

   New numerator for (3/4) = (8 ÷ 4) × 3 = 6

   New denominator for both fractions = 8

   The problem is now restated as $\frac{7}{8} \quad \frac{6}{8} \quad \frac{1}{8}$

2. $(6/9) \div (4/5) = (6/9) \times (5/4) = \frac{6}{9} \times \frac{5}{4} = \frac{30}{36}$

PROBLEM-SOLVING STRATEGIES

Problem solving is basically the processes taken in order to solve a problem. Less complex mathematical problems require fewer steps in order to find a solution. Progressively, more complex mathematical problems require several steps in order to find a solution.

Solving Word Problems

All types of mathematical problems can involve word problems. Many word problems can be solved by breaking the problem down into smaller pieces and solving for each piece. The following is a general sequence of steps used to solve word problems:

1. **Determine what the problem is asking for.** Word problems can be especially frustrating at times mainly because it is difficult to ascertain just what they are asking.
2. **Identify all of the variables.** Determine the known and unknown variables. By determining the knowns and unknowns, any superfluous information (unnecessary information not relevant to the problem) can then be eliminated.
3. **Assign the symbols.** Assign symbols to variables that may be required in some formulas.
4. **Set up the formula or list the procedure.** With the given information, set up all possible formulas or list all possible procedures that will solve the problem.

5. **Substitute the symbols.** Substitute any symbols in the formula with all relevant variable information.
6. **Solve the formula.** Be consistent with the use of units. For example, change inches to feet or feet to yards, and so on.
7. **Is the problem answered? Is the answer reasonable?**

In Practice

Using word problem techniques, solve the following:

What is the capitalization rate if a property sold for $500,000 and had a net operating income of $60,000?

Solution:

1. **Determine what the problem is asking for:** Capitalization rate
2. **Identify all of the variables:**

Net operating income = $60,000

Value = $500,000

Capitalization rate = ?

3. **Assign the symbols:**

I = Net operating income = $60,000

V = Value = $500,000

R = ?

4. **Set up the formula or list the procedure:** The income capitalization formula (IRV) is expressed three ways:

$$I = R \times V \quad \text{or} \quad R = I \div V \quad \text{or} \quad V = I \div R$$

The unknown variable determines which formula to use. In this case, we are solving for the capitalization rate (R); therefore, the middle formula is used.

5. **Substitute the symbols:**

R = $60,000 ÷ $500,000

6. **Solve the formula:**

R = $60,000 ÷ $500,000 = 0.12 or 12%

7. **Is the problem answered? Is the answer reasonable?**

Solving Equations/ Algebraic Expressions

In mathematics, equations are statements indicating the equality of two quantities (*numerical equation*) or statements using symbols (*algebraic expressions*). For example: $2 + 2 = 4$ is a numerical equation, while $x + 6 = 8$ is an algebraic expression. The right side of an equation is equal to the left side separated by an equal sign (=). Solving an equation involves finding the value or values of a variable that satisfy the statement. Algebraic operations can be used to solve equations with numbers or symbols. There are several ways in which equations can be manipulated while still maintaining a true statement:

- **Rule 1**—Multiplying or dividing both sides of the equation by the same number or variable does not change the solution.
- **Rule 2**—Adding or subtracting both sides of the equation by the same number or variable does not change the solution.
- **Rule 3**—Raising both sides of the equation by the same exponent or taking the same root of both sides of the equation does not change the solution.

These rules, applied to true statements, can manipulate equations in order to solve them. The values of the variables making the equation true are referred to as *solutions*.

To solve equations with one variable:

- Add, subtract, multiply, or divide both sides of the equation by the same number.
- Once the variable is isolated on one side of the equation, the other side of the equation is the solution.

For example, in order to solve $x + 6 = 8$, we can isolate the unknown variable, x, and solve:

- Subtract 6 from both sides, of the equation:

$$x + 6 - \mathbf{(6)} = 8 - \mathbf{6}$$

$$x + 0 = 2$$

$$x = 2$$

In Practice

Solve the equation:

$$\mathbf{3}x + \mathbf{2} = \mathbf{20}$$

Solution:

To isolate the *x*, first subtract 2 from both sides of the equation.

$$3x + 2 - 2 = 20 - 2$$

Which simplifies to:

$$3x = 18$$

Next divide both sides by 3:

$$\frac{3x}{3} = \frac{18}{3}$$

The value of the variable is now the solution:

$$x = 6$$

Interpolation and Extrapolation

Interpolation is a prediction made *between* known values of data. **Extrapolation** is any prediction made *beyond* known values of data.

For example, if a cost guide gives the per square foot costs to build a 2,200-square-foot home and a 2,400-square-foot home, but your subject is 2,300 square feet, you will need to interpolate the per square foot costs for the subject.

Good Quality

2,200 s.f.	2,400 s.f.
$92.00/s.f.	$88.00/s.f.

The difference in square feet between the two given values is 200 square feet (2,400 – 2,200). The difference in the dollars per square foot is $4.00 ($92.00 – $88.00). It is obvious that the larger the building, the lower the cost to build is per square foot.

The difference in size, 200 square feet, represents two 100-square-foot increments. If we divide $4.00 by 2 we get the incremental costs per 100 square feet of size ($4.00 ÷ 2 = $2.00).

The subject is 2,300 square feet, which represents 1 scale distance of 100 feet from 2,400 square feet. Remembering that the larger the building is the less it costs per square foot, we now can add $2.00 to the per square foot costs at 2,400 square feet.

This results in an estimated cost per square foot of $90.00 ($88.00 + $2.00).

Therefore, we can estimate that the cost to build a 2,300-square-foot building is $207,000 (2,300 sq. ft. × $90.00).

The same concept can be applied if the size of the subject was out of the known data range; in other words, larger than 2,400 square feet. If this were the case, extrapolation would be performed. Extrapolation is more risky than interpolation because it must be assumed that the same pattern will continue past the known data range.

LESSON 1 REVIEW QUESTIONS

1. $(8 + 1)^2 =$
2. $3 + (+9) =$
3. $10 - (-3) =$
4. $\{18 + [(12 - 2) \div 5]\} =$
5. $20 \times 4 \div 2 =$
6. $10 \div 5 \times 6 =$
7. $2{,}500 \div (-500) =$
8. $3 - 10 - 6 =$
9. What is 20% of 600?
10. A property sold for $300,000, which was 4.8% less than the list price. What was the property listed for, rounded to the nearest thousand?
11. The number of home listings for sale in a certain market area was 1,403 in 2004 and 1,104 in 2007. What was the percentage decrease?
12. Round 125 to the nearest ten.
13. Round 3.8099 to the nearest one-hundredth.
14. $17^1 =$
15. $87^0 =$
16. $(1/4)^4 =$
17. What is the reciprocal of 35?
18. What is the reciprocal of 1/23?
19. $(8/5) + (6/4) =$
20. $2x - 10 = 10, x =$

LESSON 1 REVIEW ANSWERS

1. $(8 + 1)^2 = (9)^2 = 81$
2. $3 + (+9) = 3 + 9 = 12$
3. $10 - (-3) =$

   $+10 - (-3) =$

   $+(10 + 3) =$

   13
4. $\{18 + [(12 - 2) \div 5]\}$

   $\{18 + [(10) \div 5]\}$

   $\{18 + [2]\} = 20$
5. $20 \times 4 \div 2 = 80 \div 2 = 40$
6. $10 \div 5 \times 6 = (10 \div 5) \times 6 = 2 \times 6 = 12$
7. $2{,}500 \div (-500) = -5$
8. $3 - 10 - 6 =$

   $+3 - 10 - 6$

   $+(3) - (10 + 6)$

   $+(3) - (16)$

   $-(16 - 3) = -(13) = -13$
9. $20\% \times 600 = ?$

   $(20 \div 100) \times 600 = 0.20 \times 600 = 120$
10. $300,000 is the result after the list price was reduced by 4.8%; therefore, it represents 95.2% (100% – 4.8%) of the list price. Remember, 95.2% converted into a decimal is 0.952.

$$\textbf{Total} = \frac{\text{Part}}{\text{Percent}}$$

$$\textbf{Total} = \frac{\$300{,}000}{0.952} = \$315{,}126, \text{ rounded to } \textbf{\$315,000}$$

    Therefore the list price was $315,000.
11. Calculate the actual difference (the increase or decrease) in the number of home listings from 2006 to 2007.

$$1{,}104 - 1{,}403 = -299$$

Then, calculate the percentage decrease with respect to the original year (2004):

$$(-299 \div 1{,}403) = -0.213$$

$$(-0.213) \times 100 = -21.3\%$$

12. 130
13. 3.81
14. $17^1 = 17$
15. $87^0 = 1$
16. $(1/4)^4 = (0.25)^4 = (0.25) \times (0.25) \times (0.25) \times (0.25) = 0.003906$
17. The reciprocal of $35 = 1/35$

    proof: $35 \times 1/35 = 35/35 = 1$
18. The reciprocal of $1/23 = 23$

    proof: $1/23 \times 23 = 23/23 = 1$
19. $(8/5) + (6/4) =$

    The LCD = 20

    New numerator for $(8/5) = (20 \div 5) \times 8 = 32$

    New numerator for $(6/4) = (20 \div 4) \times 6 = 30$

    New denominator for both fractions = 20

    The problem is now restated as $\frac{32}{20} + \frac{30}{20} = \frac{62}{20}$
20. $2x - 10 = 10$

$$2x - 10 + \mathbf{10} = 10 + \mathbf{10}$$

$$2x = 20$$

$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

LESSON 2: Calculating Area

AREA AND VOLUME

As part of the specific and general data collection process, appraisers need to calculate the area (and volume where applicable) of buildings as well as sites.

A review of basic geometric formulas is in order to calculate *area* and *volume*. **Area** is the amount of space within a two-dimensional surface enclosed by a boundary. **Volume** is the amount of space within a three-dimensional object expressed in cubic units (cubic feet, inches, or yards).

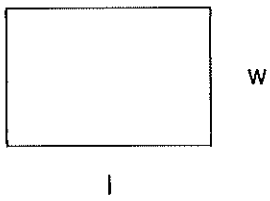
Although a data source or builder may provide area information, the appraiser must verify that the information is correct.

The area of a site is usually expressed in terms of square feet (sq. ft.) or acres, whereas the area and volume of a building is generally expressed in terms of square feet (sq. ft.) or cubic feet (cu. ft.), respectively.

The symbol for inches is " and the symbol for feet is '. For example, 55' indicates 55 feet, while 67" indicates 67 inches.

Area

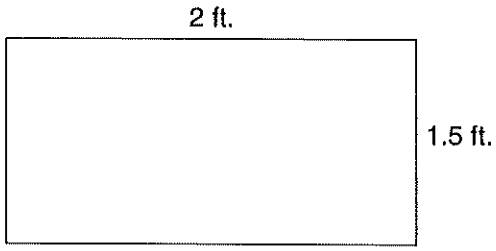
Rectangle A rectangle is a figure that has pairs of opposite sides with equal length. The area of a rectangle is the product of its length and its width. All four sides meet at right angles (90 degrees) to each other.



Area = length × width

$A = l \times w$

For example, the area of a rectangle with a length of 2 feet and a width of 1.5 feet, calculates to an area of 3 square feet (2'× 1.5').

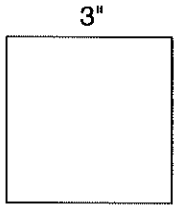


Square A square is a rectangle with four sides of equal length that meet at right angles (90 degrees). The area of a square is the length of a side(s) squared.

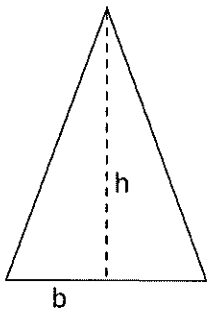
$$\text{Area} = s \times s$$

$$A = s^2$$

For example, the area of a square where the length of each of the four sides is 3 inches calculates to an area of 9 square inches (3^2).



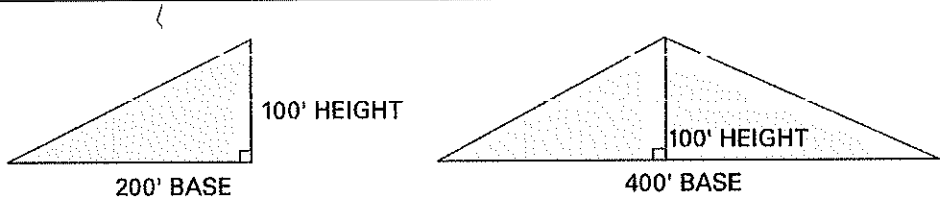
Triangle A triangle is a figure bounded by three lines that meet at three angles. The area of a triangle is equal to one-half of its base multiplied by its height. Any side of a triangle can be considered as the base; however, the height must be an imaginary line perpendicular (at a 90 degree angle) to the base. (See Figure A.1.)



$$\text{Area} = \tfrac{1}{2}(\text{base} \times \text{height})$$

$$A = \tfrac{1}{2}bh$$

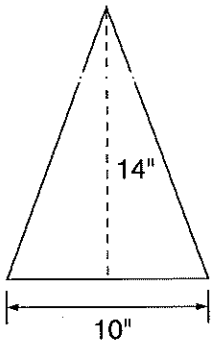
FIGURE A.1
Calculating the Area of a Triangle



0.5 x 200 (Base) x 100 (Height) = 10,000 Square Feet

0.5 x 400 (Base) x 100 (Height) = 20,000 Square Feet

For example, the area of a triangle where the base is 10" and the height is 14", calculates to an area of 70 square inches: ½ × (10"× 14").



Irregular Figures When the shape of a building or site is *irregular* (not an exact rectangle or triangle), calculate the area by the process of *decomposition*. **Decomposition** is the act of breaking down something into smaller parts. Break down irregular figures into a series of rectangles and triangles; then add together the calculated areas of each rectangle or triangle to obtain the total area. See Figure A.2 for an example of decomposition.

Volume

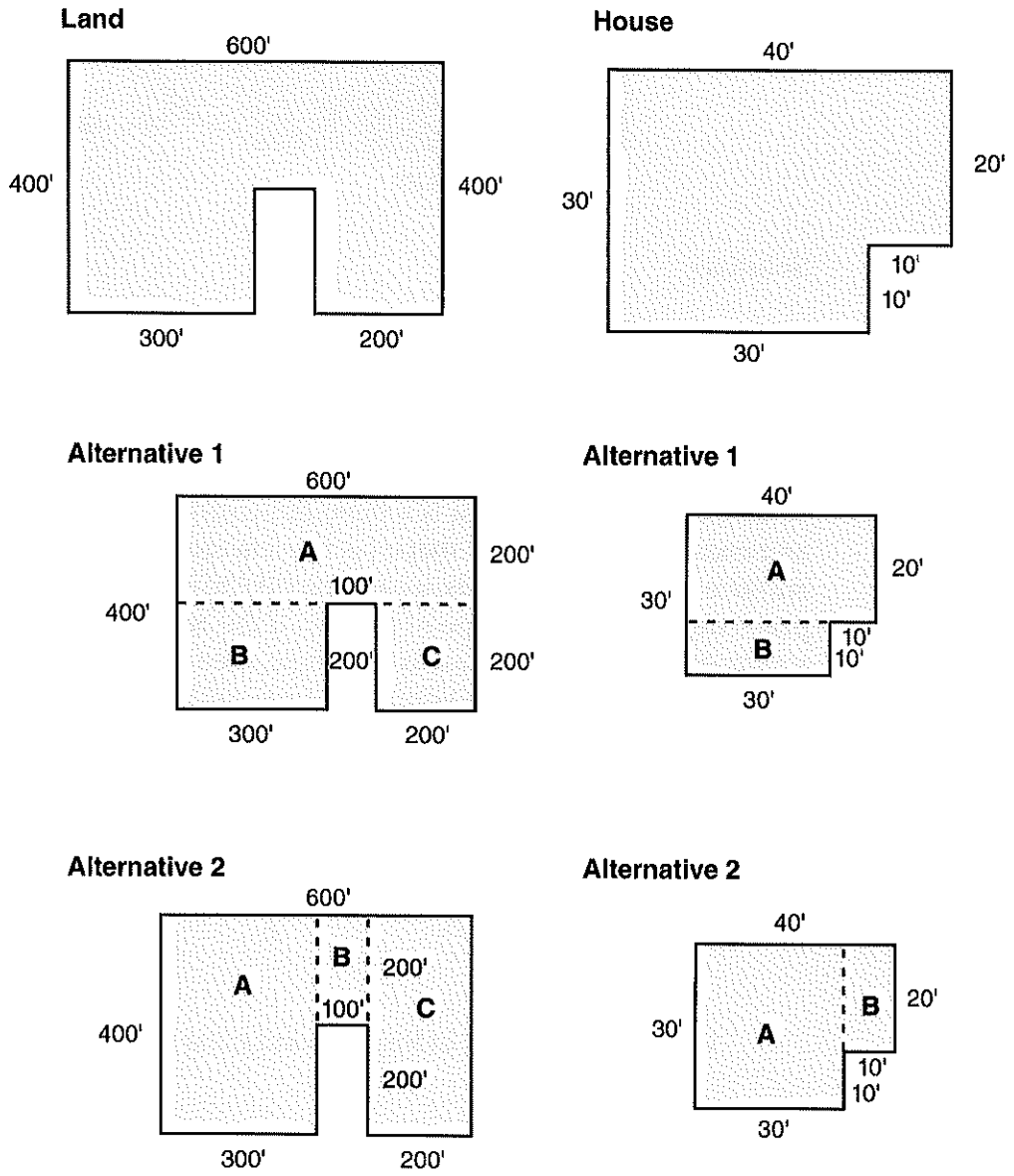
Volume is the amount of space within a three-dimensional object expressed in ***cubic units*** (cubic feet, inches, or yards) and is used to determine the capacity that an object occupies. For example, an appraiser may need to calculate volume when an appraisal assignment involves a warehouse that houses large equipment. (See Figure A.3.)

Calculate the volume of a rectangular box by multiplying the surface area of one side by the height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$V = lwh$$

FIGURE A.2
Calculating Total Area of Land and/or a House



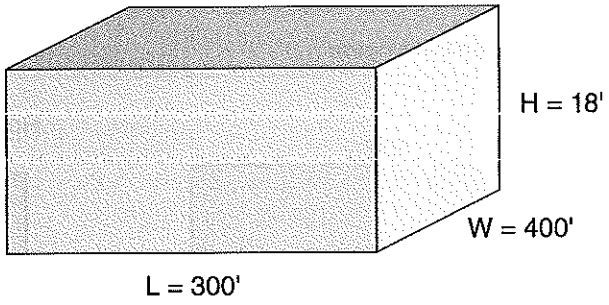
■ In Practice

Calculate the volume in cubic feet of a solid rectangle (rectangular prism) with the following dimensions:

length = 300'

width = 400'

height = 18'

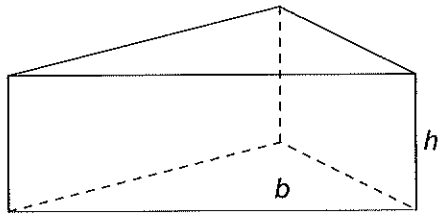


Solution:

Simply multiply the length by the width by the height (lwh).

Volume = 300' × 400' × 18' = 2,160,000 cubic feet

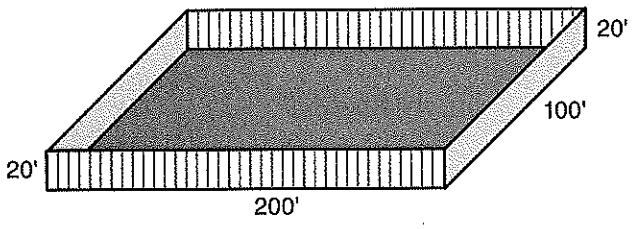
Volume of a Prism A **prism** is a three-dimensional solid figure with bases and ends that are equal in shape (as in rectangular, circular, or triangular) and size. The bases and ends are also parallel to each other. The name of the prism depends on the shape of its base. If the base is a rectangle, then it is a *rectangular prism;* if the base is a triangle, then it is a *triangular prism.* The formula for calculating the volume of a triangular prism is:



V = ½(base × height of line perpendicular to the base × width)

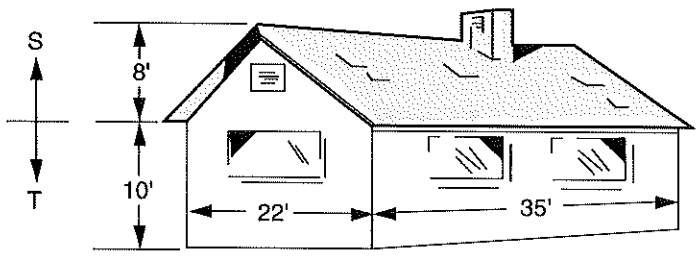
V = ½(bhw)

FIGURE A.3
Calculating Volume



For example, a home is a combination of a rectangular prism and a triangular prism. To calculate the volume of a home, including attic area, divide the building into two different figures (decomposition). Then add together the two calculated volumes for each figure, resulting in the total building volume.

For example, to compute the volume of the following house, first divide the house into two shapes, S and T.



Find the volume of S.

$$V = \frac{1}{2}(b \times h \times w) = \frac{1}{2} \times (22' \times 8' \times 35') =$$

$$\frac{1}{2}(6{,}160 \text{ cu. ft.}) = 3{,}080 \text{ cu. ft.}$$

Find the volume of T.

$$V = 22' \times 35' \times 10' = 7{,}700 \text{ cu. ft.}$$

Total volumes S and T.

$$3{,}080 \text{ cu. ft.} + 7{,}700 \text{ cu. ft.} = 10{,}780 \text{ cu. ft.}$$

Perimeter

The **perimeter** is the length of the boundary lines of a closed figure, generally expressed in terms of linear inches, linear feet, or linear yards. A **linear measurement**, also referred to as *lineal measurement*, is a single dimension relating to a line. Perimeter calculations come in handy when using cost manuals/software to determine reproduction and replacement costs used in the cost approach.

$$\text{Perimeter of a rectangle} = 2 \times \text{length} + 2 \times \text{width}$$

$$P = 2l + 2w$$

For example, the perimeter of a rectangle with a length of 20 feet and a width of 15 feet is 70 feet.

$$P = (2 \times 20) + (2 \times 15)$$

$$P = 40 + 30 = 70 \text{ feet}$$

The perimeter of a triangle is equal to the sum of the lengths of all three sides.

$$P = a + b + c$$

For example, the perimeter of a triangle with lengths of 7, 3, and 10 feet is 20 feet.

$$P = 7 + 3 + 10$$

$$= 20 \text{ feet}$$

Front Footage/Frontage

Front footage or **frontage** refers to the portion of land that faces a desirable feature, such as a street (access), a golf course, or a body of water. The length of frontage is a linear measurement expressed as front feet. A **front foot** is a strip of land 1 linear foot long. Calculating frontage is important when valuing retail properties or industrial properties as there may be a relationship between sales price and street access or a railroad siding.

Site dimensions include width and depth. The **width** of a site is length from side to side, while the **depth** is the length from front to back.

Generally the side that faces the street is the first dimension listed; therefore, the width of a site is generally the frontage. For example, a site with dimensions of 160' × 400' has a width of 160', a depth of 400', and frontage of 160'.

Like Units of Measure

When calculating areas of a shape that has different units of measure (for example, inches, feet, and yards), it is important to convert all measurements into one unit of measure. To calculate areas properly, multiply feet by feet, multiply inches by inches, and multiply yards by yards. See Figure A.4 for a conversion chart.

FIGURE A.4
Conversion Chart for Inches, Feet, and Yards

12 inches = 1 foot 36 inches = 1 yard 3 feet = 1 yard	
To convert feet to inches, *multiply the number of feet by 12.*	(ft. × 12 = in.)
To convert inches to feet, *divide the number of inches by 12.*	(in. ÷ 12 = ft.)
To convert yards to feet, *multiply the number of yards by 3.*	(yd. × 3 = ft.)
To convert feet to yards, *divide the number of feet by 3.*	(ft. ÷ 3 = yd.)
To convert yards to inches, *multiply the number of yards by 36.*	(yd. × 36 = in.)
To convert inches to yards, *divide the number of inches by 36.*	(in. ÷ 36 = yd.)

FIGURE A.5
Conversion Chart for Square Inches, Square Feet, and Square Yards

To convert square feet to square inches, *multiply the number of square feet by 144.*	(sq. ft. × 144 = sq. in.)
To convert square inches to square feet, *divide the number of square inches by 144.*	(sq. in. ÷ 144 = sq. ft.)
To convert square yards to square feet, *multiply the number of square yards by 9.*	(sq. yd. × 9 = sq. ft.)
To convert square feet to square yards, *divide the number of square feet by 9.*	(sq. ft. ÷ 9 = sq. yd.)
To convert square yards to square inches, *multiply the number of square yards by 1,290.*	(sq. yd. × 1,290 = sq. in.)
To convert square inches to square yards, *divide the number of square inches by 1,290.*	(sq. in. ÷ 1,290 = sq. yd.)

■ In Practice

1. 6' × 5" = _____ square feet or _____ square inches
2. 1 yard × 5' = _____ square feet or _____ square yards

Solution:

1. 6' × 5" = **2.5 square feet** [6 × (5 ÷ 12)] square feet or
   360 square inches [(6 × 12) × 5] square inches
2. 1 yard × 5' = **15 square feet** [(1 × 3) × 5] square feet or
   1.67 square yards [1 × (5 ÷ 3)] square yards

The calculations to convert square inches, square feet, and square yards units of measure may also be necessary. For example, there are 144 square inches in a square foot (12" × 12"). See the conversion chart in Figure A.5.

■ In Practice

1. 7,200 square inches = _____ square feet
2. 243 square feet = _____ square yards

Solution:

1. 7,200 square inches = **50** square feet; (7,200 ÷ 144) square feet
2. 243 square feet = **27** square yards; (243 ÷ 9) square yards

The calculations to convert cubic inches, cubic feet, and cubic yards units of measure may also be necessary. For example, there are 1,728 cubic inches in a cubic foot (144" × 12"). See the conversion chart in Figure A.6.

FIGURE A.6
Conversion Chart for Cubic Inches, Cubic Feet, and Cubic Yards

To convert cubic feet to cubic inches, *multiply the number of cubic feet by 1,728.*	(cu. ft. × 1,728 = cu. in.)
To convert cubic inches to cubic feet, *divide the number of cubic inches by 1,728.*	(cu. in. ÷ 1,728 = cu. ft.)
To convert cubic yards to cubic feet, *multiply the number of cubic yards by 27.*	(cu. yd. × 27 = cu. ft.)
To convert cubic feet to cubic yards, *divide the number of cubic feet by 27.*	(cu. ft. ÷ 27 = cu. yd.)
To convert cubic yards to cubic inches, *multiply the number of cubic yards by 46,656.*	(cu. yd. × 46,656 = cu. in.)
To convert cubic inches to cubic yards, *divide the number of cubic inches by 46,656.*	(cu. in. ÷ 46,656 = cu. yd.)

■ In Practice

1. 8,640 cubic inches = _____ cubic feet
2. 1,620 cubic feet = _____ cubic yards

Solution:

1. 8,640 cubic inches = **5** cubic feet; (8,640 ÷ 1,728) cubic feet
2. 1,620 cubic feet = **60** cubic yards; (1,620 ÷ 27) cubic yards

Unit of Comparison

A **unit of comparison** is a per unit indicator. Within a specific market, the appraiser must analyze information gathered during the general data collection process and determine which unit or units of comparison accurately depict the expectations of the market participants. For example, use front footage when market data indicates a relationship between sales price and frontage. The use of statistics helps determine which unit of comparison to employ by using various indicators.

Common units of comparison for various property types follow:

- **Units of Comparison for Vacant Land**
  - Agricultural—price per acre, animal unit
  - Residential—price per acre, square foot
  - Multifamily—price per potential unit, square foot, acre
  - Commercial—price per acre, square foot, front foot
  - Industrial—price per acre, square foot, front foot
- **Units of Comparison for Improvements**
  - Farm—price per animal unit
  - Residential—price per room, bedroom, square foot
  - Multifamily—price per unit, room, bedroom, bath, square foot

- Commercial—price per square foot, unit
- Industrial—price per square foot, cubic foot

The preceding is a list of common units of comparison. For any given property type in any given market, there may be a relationship between sales price and a unit of comparison not listed.

■ **In Practice**

What is the purchase price per square foot of a vacant site with dimensions of 50' × 180', and a purchase price of $250,000?

Solution:

area of vacant site = 9,000 square feet (50' × 180')

purchase price per square foot = purchase price ÷ square footage of vacant site

$250,000 ÷ 9,000 = $28 per square foot, *rounded*

■ **In Practice**

Using the preceding information from the In Practice problem, calculate the purchase price per acre and per front foot. Hint: An acre contains 43,560 square feet.

Solution:

area of vacant site = 9,000 square feet (50' × 180')

acreage of vacant site = 9,000 ÷ 43,560 = 0.21 acre

purchase price ÷ acreage of vacant site = purchase price per acre

$250,000 ÷ 0.21 acre = $1,190,476 per acre

frontage of vacant site 50' × 180' site = 50'

purchase price ÷ frontage = purchase price per front foot

$250,000 ÷ 50' = $5,000 per front foot

Gross Living Area (GLA)

One of the most important aspects of the specific data collected about a property is the gross living area (GLA). **Gross living area** of a house is the *above grade area* enclosed by the outside walls. **Above grade area** is the portion of a building that is above ground level. Below grade areas include basements and subbasements. Also not included in GLA are areas of a

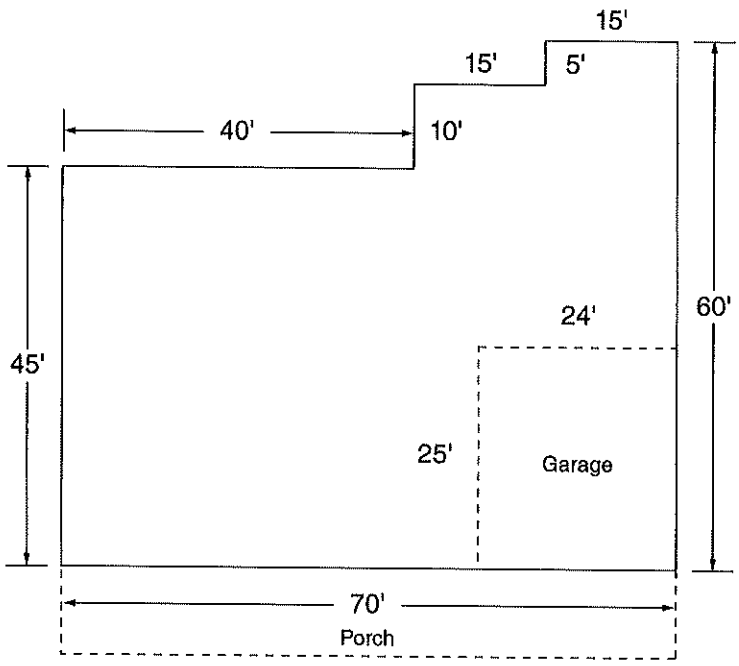
building that are not heated or cooled, or areas that are not actually "livable." These areas include garages, open porches, unfinished attics, and the second-floor area that does not actually have a floor in a two-story foyer entrance.

As mentioned earlier, a data source or builder may provide the square footage of land and gross living area information; however, the appraiser must verify that the information is correct. The use of proper measuring devices and techniques is extremely important. There are many measuring devices on the market to aid appraisers. The following steps help ensure the proper calculation of GLA:

- Draw a sketch of the footprint (foundation) of the building.
- Measure and record all the outside measurements.
- Measure the garage, whether attached or unattached. If it is an attached garage, the interior wall is considered an outside wall of the building.
- Measure any attached porches. The interior wall of a porch is considered to be an outside wall of the building.
- If the building has more than one story, check each story against the footprint of the building. Be sure to account for overhang areas (i.e., second-story areas cantilevered over the first floor). Conversely, subtract any notched-out areas in the second floor.
- Convert any measurements in inches to feet. For example, convert 12'6" to 12.5'.
- Double-check measurements for reasonableness and accuracy, and take many photos if necessary.
- Use decomposition to calculate areas of all floors. Do not include below-grade areas (basements or subbasements), garages, unheated and cooled areas, porches, and so on.

■ In Practice

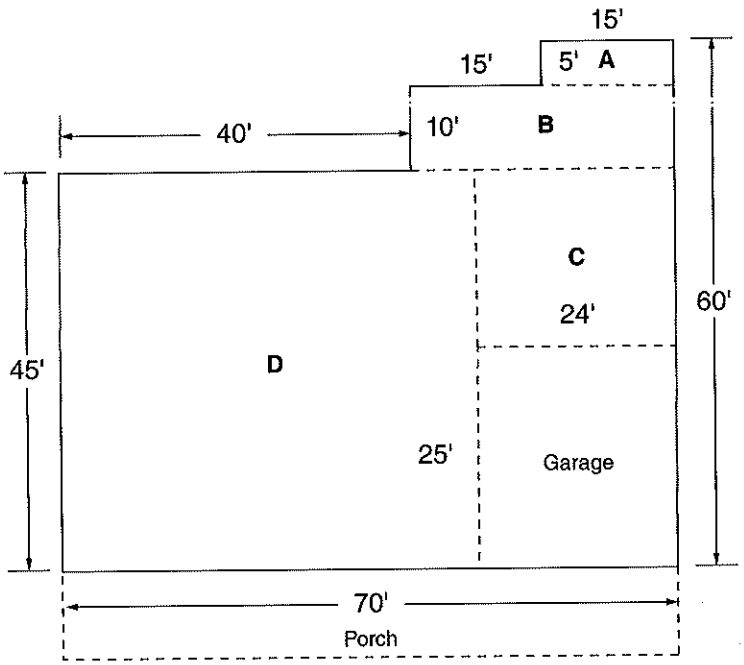
Calculate the gross living area of the single-story home shown below.



Solution:

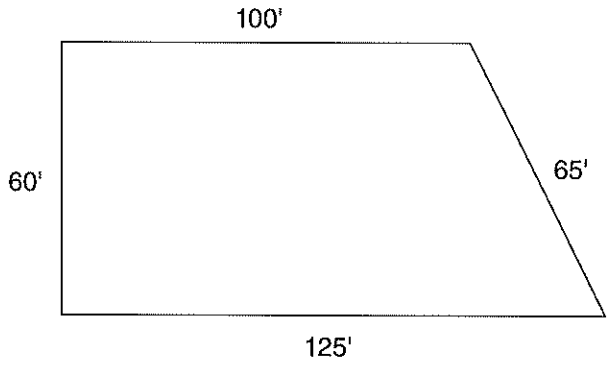
GLA:

A = 5' × 15' =	75 sq. ft.
B = 10' × (15' + 15') = 10' × 30' =	300 sq. ft.
C = 24' × (45' – 25') = 24' × 20' =	480 sq. ft.
D = (70' – 24') × 45' = 46' × 45' =	2,070 sq. ft.
Total =	**2,925 sq. ft.**

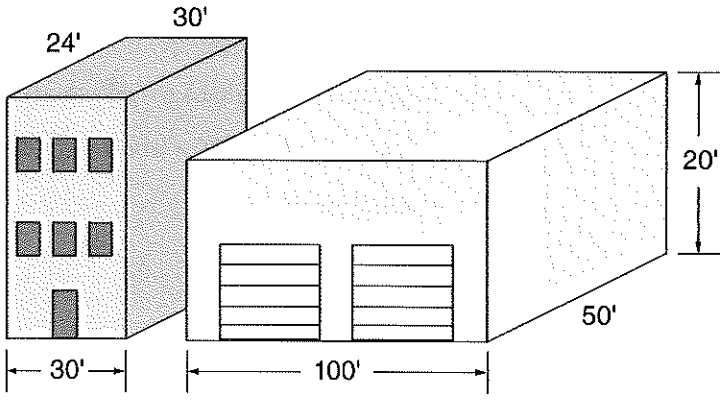


LESSON 2 REVIEW QUESTIONS

1. 16" × 8" = _____ square feet or _____ square inches
2. 86 square yards = _____ square feet
3. 3 yards × 2' = _____ square feet or _____ square yards
4. Calculate the area of a triangle with a base of 40" and a height of 15".
5. Compute the area of the following figure:



Use the figure below to answer questions 6, 7, and 8.



6. What is the cubic area of the garage space?
7. What is the square-foot area of the two-story office area?
8. What is the total building area in square feet?

LESSON 2 REVIEW ANSWERS

1. 16' × 8" = **10.67 square feet;** [16 × (8 ÷ 12)] square feet
   or **1,536 square inches;** (16 × 12) × 8 square inches
2. 86 square yards = **774 square feet;** (86 × 9) square feet
3. 3 yards × 2' = **18 square feet;** [(3 × 3) × 2] square feet
   or **2 square yards;** [3× (2 ÷ 3)] square yards
4. A= ½(bh) = ½ × (40" × 15") = ½ × (600 sq. in) = 300 sq. in.
5. Area of rectangle = l × w = 100' × 60' = 6,000 sq. ft.
   Area of triangle = ½(bh) = ½(125' – 100')(60') = ½(25' × 60')= ½(1,500 sq. ft.) = 750 sq. ft.
   Total area = 6,000 sq. ft. + 750 sq. ft. = 6,750 sq. ft.
6. Volume of rectangular prism (garage) = lwh
   = 50' × 100' × 20'= 100,000 cubic feet
7. Area of rectangle (office area) = l × w = 24' × 30'
   = 720 sq. ft. × 2 floors = 1,440 square feet
8. Total area of building = office area + garage area
   = 1,440 sq. ft. + (50' × 100') = 1,440 sq. ft. + 5,000 sq. ft.
   = 6,440 sq. ft.

LESSON 3: Statistics Overview

WHAT IS STATISTICS?

Statistics is the mathematics of collecting, classifying, and interpreting numerical data to determine if certain values represent characteristics of a population. In statistics, a **population** is a set of measurements that describes some phenomenon. A population could be a group of individuals, objects, or items. For example, a population can be all single-family homes located within a certain town or village that have a certain *parameter.*

A **parameter** is a characteristic of a population. For example, an appraiser might be interested in the average sales price of all *two-bedroom, 1,600-square-foot* (the parameter) single-family homes. Because most populations are too large to observe, a *sample* must be taken for statistical measurement.

A **sample** is a subset of measurements selected from a population. A **statistic** is a characteristic of a sample.

Statistics enables one to make an educated guess about a population parameter (the sales price of all two-bedroom 1,600-square-foot single-family homes), based on the statistic computed from a sample. It is important that the sample chosen is typical of the population. In other words, the sample must be representative of the population.

MEASURES OF CENTRAL TENDENCY

Measures of central tendency are numbers that "cluster" around the "middle" of a set of values. For example, the central tendency may be the typical sales price of a single-family home.

Three common measures of central tendency, the "middle" numbers, are the *mean, median,* and *mode.* All three measures of central tendency can identify the typical phenomena in a population or sample. Discussions of all three follow.

A **data set** is a set of data, which are observations, or measurements of a specific variable. Data sets must be arranged in an orderly fashion before calculating various measures of central tendency, namely, the median and

the mode. Data arranged in order of size is called an **array.** In other words, the arrangement of data is in increasing or decreasing order. The following is an example of an array using the sales prices of seven single-family homes.

$450,000

$395,000

$390,000

$390,000

$389,000

$380,000

$375,000

Note that the sales prices are in descending order.

Mean

The most familiar and basic statistical measure is the arithmetic **mean,** which is an average value for a group of numerical observations. The mean is the sum of all data values divided by the total number of data values. The typical denotation for the mean of a sample is $\bar{x}$ (read as *X* bar).

The formula for the sample mean is usually denoted as follows:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

or

$$\bar{x} = \frac{X_1 + X_2 + X_3 + X_4 + \ldots X_N}{N}$$

Where, N = number of observations

X = data value

Σ = *the summation of*

■ **In Practice**

Calculate the mean of the following data set:

{4, 9, 6, 8}

Solution:

The number of observations is 4 because there are four numbers.

$$\textbf{mean} = \bar{x} = \frac{4+9+6+8}{4} = 6.75$$

Median

The **median** is the "middle value" in an array of numbers. This middle value has an equal number of values above and below it. In other words, half of the array will have values less than or equal to the median and half of the array will have values equal to or greater than the median. To find the median of a data set, arrange all the observations from lowest value to highest value or from highest value to lowest value, creating an array, and pick the middle one. If there is an even number of observations, calculate the mean of the two middle values.

For example: The median of the previously described array is $390,000.

$450,000

$395,000

$390,000

→ **$390,000**

$389,000

$380,000

$375,000

Adding an observation of $370,000 to this array changes the median to $389,500.

$450,000

$395,000

$390,000

→ $390,000
→ $389,000
} $\frac{\$390,000 + \$389,000}{2} = \$389,500$

$380,000

$375,000

$370,000

Mode

The **mode** is the most frequently occurring number in an array. There is no formula to learn; simply choose the number that appears most frequently in the data set or array. For example, the mode of the following array is $390,000.

$450,000

$395,000

→ **$390,000**

→ **$390,000**

$389,000

$380,000

$375,000

$370,000

In this array, $390,000 appears twice, while the other values only appear once.

If a data set has two modes, it is called a *bimodal* data set. Some data sets do not have modes. For example, the following data set does not have a mode because each data value only appears once.

{65, 75, 98, 101, 200}

■ In Practice

Calculate the mean, median, and mode of the following data set, which represents the annual rent per square foot of a sample of office building rents.

{$23, $19, $18, $24, $25, $20, $23}

Solution:

- First, arrange the data set into an array (arranging in descending or ascending order).

  1. $18
  2. $19
  3. $20
  4. $23
  5. $23
  6. $24
  7. $25

- Then, calculate the mean:

$$\textbf{mean} = \bar{x} = \frac{18 + 19 + 20 + 23 + 23 + 24 + 25}{7} = 21.71$$

The mean is, therefore, $22 annual rent per square foot, *rounded.*

- Next, find the median:

  $18
  $19
  $20
→ **$23**
  $23
  $24
  $25

The median is the middle value. In this case, the median is $23 annual rent per square foot.

- Finally, find the mode:

  $18
  $19
  $20
→ **$23**
→ **$23**
  $24
  $25

The mode is $23 annual rent per square foot because it appears twice.

Notice that the mean is $22, the median is $23, and the mode is $23. When studying the distribution of all three measures of central tendency, whether or not they are all equal in value has great significance. However, in this lesson, the discussion is limited to the definition and the calculation of these measures of central tendency.

Selecting a Measure of Central Tendency

It is prudent to review the strength and weakness of each measure of central tendency before selecting a measure to represent a population. For example, while the mean is the most frequently used measure, it does have its drawbacks.

- Mean vs. Median and Mode

  The mean is sometimes preferred because it uses every observation in the data set; however, using the median or mode is more accurate when there are extreme values (outliers) in the data set. Using the mean when there are outliers distorts what should be considered a typical result. The median and the mode are insensitive to outliers. However, there is a risk when using the mode, if the mode happens to be an outlier.

- Mode vs. Mean and Median

  While the mode is the weakest measurement, it makes sense for **nominal data**, which is data that does not consist of numerical values. For example, taking a sample of American names for baby boys, the name that appears most frequently may be Michael. The mode would then be the name Michael.

  Another appropriate use of the mode is to paint a more realistic picture. For example, a sample consisting of the number of bedrooms per single-family home may have a mean of 2.5, a median of 3.5, and a mode of 3. Using the three bedrooms as representative of what is typical is more desirable than using 2.5 bedrooms or 3.5 bedrooms.

MEASURES OF VARIABILITY

In addition to the measures of central tendency, which only uncover the center of distribution, other measures are used to describe data. *Variability* refers to how individual values in a data set differ from each other. **Measures of variability** measure how "spread out" values in a data set are, in other words, whether or not values "cluster" around the mean or median or are scattered. Three frequently used measures of variability are *range, variance,* and *standard deviation.*

Range

The simplest and most obvious measure of variation is the *range*. **Range** is simply the difference between the largest and the smallest values in a data set. However, range only measures the most varied values of the data set.

For example, the following is an array indicating sales prices of seven two-bedroom homes:

$250,000

$200,000

$192,000

$190,000

$188,000

$185,000

$160,000

The highest sales price is $250,000 and the lowest is $160,000; therefore, the range is **$90,000** ($250,000 – $160,000).

■ In Practice

Determine the range of the following data set:

{56, 44, 63, 60, 42, 52, 56, 51}

Solution:

First arrange the data set into an array.

42

44

51

52

56

56

60

63

The highest number is 63 and the lowest number is 42, so the range is 21 (63 – 42).

Variance

Variance is a measure of how spread out a distribution is, a measure of the dispersion of a data set around the mean. To calculate the variance, calculate the sum of the squares of the differences (deviations) from the mean, and then divide by the total number of observations minus 1.

$$\text{Variance} = s^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}$$

Where, N = number of observations

X = data value

Σ = *the summation of*

$\bar{x}$ = sample mean

Note that the $N - 1$ in the denominator is used when computing the *sample* variance. When computing the *population* variance, use N alone in the denominator.

Computing the variance by dividing by N, instead of $N - 1$, gives an underestimate of the population variance because the sample mean is an estimate of the population mean, which is unknown. In practice, for a large N, the distinction is often a minor one.

For example, using the array in the range example calculate the variance.

- First calculate the mean.

42
44
51
52
56
56
60
63
Total 424

mean = (424 ÷ 8) = 53

- Next, calculate the deviations from the mean for each observation. Square them and then total them.

Deviations from the mean	Squares of deviations from the mean
$(42 - 53) = -11$	$(42 - 53)^2 = (-11)^2 = 121$
$(44 - 53) = -9$	$(44 - 53)^2 = (-9)^2 = 81$
$(51 - 53) = -2$	$(51 - 53) = (-2)^2 = 4$
$(52 - 53) = -1$	$(52 - 53)^2 = (-1)^2 = 1$
$(56 - 53) = 3$	$(56 - 53)^2 = (3)^2 = 9$
$(56 - 53) = 3$	$(56 - 53)^2 = (3)^2 = 9$
$(60 - 53) = 7$	$(60 - 53)^2 = (7)^2 = 49$
$(63 - 53) = 10$	$(63 - 53)^2 = (10)^2 = 100$
	374

- Finally, divide the total squares of deviations from the mean by the number of observations minus 1.

$$\text{Variance} = (374 \div 7) = 53.4$$

Standard Deviation

Standard deviation is another statistical measure of the dispersion or variability in a distribution. Standard deviation is the square root of the arithmetic mean of the squares of the deviations from the sample mean. Simply put, it is the square root of the variance of a data set. It measures how spread out the values in a data set are. The more spread apart the data is, the higher the deviation, hence a less reliable sample. Another way of thinking of standard deviation is as a measure of uncertainty.

The square root of the variance provides the standard deviation ($\sqrt{s^2}$).

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

For example, the standard deviation of the previous data set is the square root of the variance = $\sqrt{s^2} = \sqrt{53.4} = 7.3$

For a single data set (i.e., a data set of only one unit of comparison) the standard deviation does not tell us much. However, comparing the standard deviation of two or more data sets (units of comparisons) can reveal

the reliability of using one unit of comparison compared to the other. In other words, if the standard deviation of the price per square foot is greater than the standard deviation of the price per front foot, this indicates that the price per front foot data values are clustered (less scattered) about their mean. This allows for a more accurate prediction of what is typical in the market (population).

Properties of Variance and Standard Deviation

The variance and the standard deviation are always positive numbers. If all of the data values in a data set are equal, there is no variability and the variance and the standard deviation will equal 0. Similar to the arithmetic mean, variance and standard deviation use every observation, including outliers.

A larger standard deviation indicates that the data values are far from the mean, while a smaller standard deviation indicates that they are clustered closely around the mean and therefore a more reliable indicator. However, in this lesson, the discussion is limited to the definition and the calculation of these measures of variability.

Weighted Average

The **weighted average** is a method of computing an arithmetic mean of a set of numbers in which some of the elements of the set carry more weight (importance) than others. Each element is assigned a weight factor depending upon its relative position to the others in the data set. Use the following steps to compute weighted averages:

1. Multiply the element by its weight
2. Add up the products of the element multiplied by its weight to arrive at total value
3. Add the weights themselves to get a total weight
4. Divide the total value by the total weight

For example, suppose a data set contains the final adjusted sales price of three comparable sales:

Sale 1:	$330,000
Sale 2:	$310,000
Sale 3:	$308,000

While performing the sales comparison approach, the appraiser may feel that comparable sale 1 was the *most* similar to the subject, requiring the least amount of adjustments. That appraiser would then give the greatest weight to sale 1.

Sale 1:	$330,000 × 10 =	$3,300,000
Sale 2:	$310,000 × 5 =	$1,550,000
Sale 3:	$308,000 × 5 =	$1,540,000
Total	20	$6,390,000
Weighted average = $6,390,000 ÷ 20 =		$319,500

In this example, sale 2 and sale 3 are given the same weight because they are of equal importance.

Rather than choosing an arbitrary number like 20, weight factors can be expressed as percentages as long as the aggregate of all assigned weights adds up to 100 percent. Applying percentage weights to the previous example will result in the same outcome. Note, just as in the previous example, the weight assigned to sale 1 is twice that of the weight assigned to each sale 2 and sale 3.

Sale 1:	$330,000 × 0.50 =	$165,000
Sale 2:	$310,000 × 0.25 =	$77,500
Sale 3:	$308,000 × 0.25 =	$77,000
Total		= $319,500

Rather than simply using the arithmetic mean of $316,000, the weighted average can then be used as it may be more representative of what is typical of the population (the ultimate goal). The language used in the reconciliation of the sales comparison approach might then include words like, "comparable sale 1 was given greatest weight as it is most similar to the subject..."

Appraisers use statistics to support assumptions made about a market that eventually lead to an estimate of value. The appraiser needs to choose the relevant information in order to draw valid inferences from the sample to the population being studied. The use of measures of central tendency in conjunction with measures of variability enables the appraiser to derive logical conclusions about a population (market) based on information, which is known to be true about a sample. This lesson's discussion of statistical techniques was a brief overview. A more thorough study of statistical techniques is beyond the scope of this text; however, advanced texts, courses, and software are available to today's appraiser.

LESSON 3 REVIEW QUESTIONS

1. A _____ standard deviation indicates that the data values are far from the mean, while a _____ standard deviation indicates that they are clustered closely around the mean and therefore a more reliable indicator.

2. Extreme values in a data set are referred to as
   a. ranges.
   b. outliers.
   c. means.
   d. medians.

3. Standard deviation is simply the square root of the _____ _____.

4. A(n) _____ is a data set that has been organized according to size.

5. The difference between the largest and the smallest values in a data set is the
   a. outlier.
   b. mean.
   c. median.
   d. range.

6. The range, variance, and standard deviation are measures of
   a. variance.
   b. central tendency.
   c. variability.
   d. deviations.

7. Calculate the mean, median, and mode of the following data set of a sample containing price per square foot:

   {$156, $125, $177, $124, $125, $170, $150}

8. Calculate the sample variance and sample standard deviations of the following data set:

   {11, 15, 16, 20, 23}

9. A sample is a subset of a _____.

10. A parameter is a characteristic of a _____.

LESSON 3 REVIEW ANSWERS

1. larger, smaller
2. a. outliers
3. variance
4. array
5. d. range
6. c. variability
7. $124
   125
   125
   150
   156
   170
   177
   $1,027

   mean = ($1,027 ÷ 7) = 147, *rounded*

   median = 150

   mode = 125
8. {11, 15, 16, 20, 23}, mean = (85 ÷ 5) = 17

   Squares of deviations from the mean

   $(11 - 17)^2 = (-6)^2 = 36$

   $(15 - 17)^2 = (-2)^2 = 4$

   $(16 - 17)^2 = (-1)^2 = 1$

   $(20 - 17)^2 = (3)^2 = 9$

   $(23 - 17)^2 = (6)^2 = 36$

   86

Divide the total squares of deviations from the mean by the number of observations minus 1.

Variance – (86 ÷ 4) – 21.5, Standard Deviation = $\sqrt{21.5}$ = 4.6

9. population

10. population

LESSON 4: Mathematics of Finance

TIME VALUE OF MONEY

Most appraisal assignments require the use of the income approach; therefore, appraisers need a thorough understanding of the time value of money. **Time value of money** is the core principle of finance that holds that money available at the present time is worth more than the same amount in the future. This is possible because of money's earning capacity. For example, money deposited into a savings account will earn interest. Therefore, a specific amount of money received today is more desirable than receiving that same amount of money in the future. Time value of money results from the concept of interest. *Interest* is the cost of using someone else's money. It is a charge for borrowing money, usually expressed as a percentage of the amount borrowed *(principal)*. There are two types of interest: *simple interest* and *compound interest*.

Simple Interest and Compound Interest

Simple interest is interest earned on the principal only, not including any annual accrued (accumulated) interest. For example, if you were to earn 5% annual interest on an original amount of $100, over the next five years, your interest earned would be calculated as follows:

Year 1: 5% × $100.00 = 0.05 × $100 = $5 in interest for a total of $105

Year 2: 5% × $100.00 = 0.05 × $100 = $5 in interest for a total of $110

Year 3: 5% × $100.00 = 0.05 × $100 = $5 in interest for a total of $115

Year 4: 5% × $100.00 = 0.05 × $100 = $5 in interest for a total of $120

Year 5: 5% × $100.00 = 0.05 × $100 = $5 in interest for a total of $125

Compound interest is interest computed on the original principal plus any accrued interest. For example, if you earn 5% compound interest on an original amount of $100.00, over the next five years, your interest earned would be calculated as follows:

Year 1: 5% × $100.00 = 0.05 × $100.00 = $5 in interest for a total of $105.00

Year 2: 5% × $105.00 = 0.05 × $105.00 = $5.25 in interest for a total of $110.25

Year 3: 5% × $110.25 = 0.05 × $110.25 = $5.51 in interest for a total of $115.76

Year 4: 5% × $115.76 = 0.05 × $115.76 = $5.79 in interest for a total of $121.55

Year 5: 5% × $121.55 = 0.05 × $121.55 = $6.08 in interest for a total of $127.63

From these two examples, it is obvious that at the end of year 5 the total accumulated amount is greater when computed using compound interest rather than simple interest.

The most important component of compound interest is the compounding period. The **compounding period** is the interval at which interest is actually paid. It is the length of time that elapses before interest compounds. The interval can be annually, quarterly, monthly, or daily. In the previous example, the compounding period is annually. At the end of the first interval, the interest is calculated and then added to the principal. With compound interest, at the end of all subsequent intervals, the interest is calculated on the principal plus interest from previous intervals. Compound interest may be calculated either at the beginning of each interval or at the end of each interval.

Compounding and Discounting

Compounding is the financial concept of converting today's money into an equivalent amount at some time in the future, a **future value.** The future value consists of the principal amount, plus any accrued interest. Compounding assumes that earned interest is reinvested and therefore continues to earn interest over the time period. **Discounting** is the financial concept of converting an estimated future income into a **present value** using a **discount rate.** Discounting an expected future cash flow is the basis of the income approach. Simply stated, it is the present value of a future benefit. The compounding and discounting processes involve four variables:

1. The time period
2. The interest rate
3. The future value (expected future cash flow)
4. The present value

In order to calculate any one of these variables, the other three variables need to be known.

In the preceding compound interest example, three variables were known:

1. Time period: 5 years
2. Interest rate: 5%
3. Present value: $100

Hint: For greater accuracy, set your calculator to six decimal points.

SIX FUNCTIONS OF A DOLLAR

Compound Factors and Discount Factors

There are six basic time value of money factors, known as the **six functions of a dollar.** Published tables with factors of all six functions of a dollar can be used. These tables are known as the **"Ellwood Tables"** named after the late L.W. Ellwood. Ellwood tables were based on the six functions of a dollar formulas and were widely used prior to the modern financial calculator. Looking at Figure A.7, this table has factors for a 10% annual interest rate, compounded annually (compound interval of one year) over time periods from 1 year to 50 years.

To use a published table, the dollar amount being considered, whether the amount is loaned, borrowed, or invested, is multiplied by the appropriate factor in order to arrive at the present value or future value.

The six columns contain factors for each function of a dollar. Each function of a dollar can also be expressed mathematically. Following is a discussion of all six functions of a dollar, their factors, and the associated mathematical formulas.

1. Column 1: **Future value of $1**—The future value of one dollar is the amount that $1 can grow to over a specific period of time. For example, at the end of 5 years, $1 compounded annually (at the end of each year) at a 10% annual interest rate will grow to $1.61; $1 × 1.610510. (See column 1 of the table in Figure A.7.)

   Formula: Future value of $1 = $(1 + i)^n$

   Where, i = interest rate

   n = number of time periods

   Using the same example:

   $(1 + 0.10)^5 = (1.10)^5 = 1.610510$; $1 × 1.610510 = $1.61

2. Column 2: **Future value annuity of $1 per year**—The future value of one dollar per period (*annuity*) is the amount that constant periodic investment of $1 can grow to over a specific period of time. An **annuity** is a level stream of cash flows for a fixed period of time. For example, at the end of 5 years $1 invested at the end of every year compounded annually at a 10% annual interest rate will grow to $6.11; $1 × 6.105100. (See column 2 of the table in Figure A.7.)

FIGURE A.7
Six Functions of a Dollar

10.00% Annual Interest Rate

Years	1 Future Value of $1	2 Future Value Annuity of $1 per Year	3 Sinking Fund Factor	4 Present Value of $1 (Reversion)	5 Present Value Annuity of $1 per Year	6 Payment to Amortize $1	Years
1	1.100000	1.000000	1.000000	0.909091	0.909091	1.100000	1
2	1.210000	2.100000	0.476190	0.826446	1.735537	0.576190	2
3	1.331000	3.310000	0.302115	0.751315	2.486852	0.402115	3
4	1.464100	4.641000	0.215471	0.683013	3.169865	0.315471	4
5	1.610510	6.105100	0.163797	0.620921	3.790787	0.263797	5
6	1.771561	7.715610	0.129607	0.564474	4.355261	0.229607	6
7	1.948717	9.487171	0.105405	0.513158	4.868419	0.205405	7
8	2.143589	11.435888	0.087444	0.466507	5.334926	0.187444	8
9	2.357948	13.579477	0.073641	0.424098	5.759024	0.173641	9
10	2.593742	15.937425	0.062745	0.385543	6.144567	0.162745	10
11	2.853117	18.531167	0.053963	0.350494	6.495061	0.153963	11
12	3.138428	21.384284	0.046763	0.318631	6.813692	0.146763	12
13	3.452271	24.522712	0.040779	0.289664	7.103356	0.140779	13
14	3.797498	27.974983	0.035746	0.263331	7.366687	0.135746	14
15	4.177248	31.772482	0.031474	0.239392	7.606080	0.131474	15
16	4.594973	35.949730	0.027817	0.217629	7.823709	0.127817	16
17	5.054470	40.544703	0.024664	0.197845	8.021553	0.124664	17
18	5.559917	45.599173	0.021930	0.179859	8.201412	0.121930	18
19	6.115909	51.159090	0.019547	0.163508	8.364920	0.119547	19
20	6.727500	57.274999	0.017460	0.148644	8.513564	0.117460	20
21	7.400250	64.002499	0.015624	0.135131	8.648694	0.115624	21
22	8.140275	71.402749	0.014005	0.122846	8.771540	0.114005	22
23	8.954302	79.543024	0.012572	0.111678	8.883218	0.112572	23
24	9.849733	88.497327	0.011300	0.101526	8.984744	0.111300	24
25	10.834706	98.347059	0.010168	0.092296	9.077040	0.110168	25
26	11.918177	109.181765	0.009159	0.083905	9.160945	0.109159	26
27	13.109994	121.099942	0.008258	0.076278	9.237223	0.108258	27
28	14.420994	134.209936	0.007451	0.069343	9.306567	0.107451	28
29	15.863093	148.630930	0.006728	0.063039	9.369606	0.106728	29
30	17.449402	164.494023	0.006079	0.057309	9.426914	0.106079	30
31	19.194342	181.943425	0.005496	0.052099	9.479013	0.105496	31
32	21.113777	201.137767	0.004972	0.047362	9.526376	0.104972	32
33	23.225154	222.251544	0.004499	0.043057	9.569432	0.104499	33
34	25.547670	245.476699	0.004074	0.039143	9.608575	0.104074	34
35	28.102437	271.024368	0.003690	0.035584	9.644159	0.103690	35
36	30.912681	299.126805	0.003343	0.032349	9.676508	0.103343	36
37	34.003949	330.039486	0.003030	0.029408	9.705917	0.103030	37
38	37.404343	364.043434	0.002747	0.026735	9.732651	0.102747	38
39	41.144778	401.447778	0.002491	0.024304	9.756956	0.102491	39
40	45.259256	442.592556	0.002259	0.022095	9.779051	0.102259	40
41	49.785181	487.851811	0.002050	0.020086	9.799137	0.102050	41
42	54.763699	537.636992	0.001860	0.018260	9.817397	0.101860	42
43	60.240069	592.400692	0.001688	0.016600	9.833998	0.101688	43
44	66.264076	652.640761	0.001532	0.015091	9.849089	0.101532	44
45	72.890484	718.904837	0.001391	0.013719	9.862808	0.101391	45
46	80.179532	791.795321	0.001263	0.012472	9.875280	0.101263	46
47	88.197485	871.974853	0.001147	0.011338	9.886618	0.101147	47
48	97.017234	960.172338	0.001041	0.010307	9.896926	0.101041	48
49	106.718957	1057.189572	0.000946	0.009370	9.906296	0.100946	49
50	117.390853	1163.908529	0.000859	0.008519	9.914814	0.100859	50

Formula: Future value of \$1 per period = $\frac{(1+i)^n - 1}{i}$

Where, i = interest rate

n = number of time periods

Using the same example:

$$\frac{(1+0.10)^5 - 1}{0.10} = \frac{(1.6105100) - 1}{0.10} = \frac{0.610510}{0.10}$$

$$= 6.105100 = \$1 \times 6.105100$$

$$= \$6.11$$

Where, $i = 0.10$

$n = 5$

3. Column 3: **Sinking fund factor**—The sinking fund factor represents the level percentage factor that must be invested at the end of each period in order to accumulate \$1. For example, if a lump sum of \$1 is needed at the end of the 5-year period, \$ 0.16 must be invested at a 10% annual interest rate at the end of each year for 5 years. Calculate this by multiplying \$1 by the sinking fund factor of 0.163797. (See column 3 of the table in Figure A.7.)

Formula: Sinking fund factor = $\frac{i}{(1+i)^n - 1}$

Where, i = interest rate

n = number of time periods

Using the same example:

$$\frac{0.10}{(1+0.10)^5 - 1} = \frac{0.10}{(1.610510) - 1} = \frac{0.10}{0.610510}$$

$$= 0.163797$$

$$= \$1 \times 0.163797 = \$0.16$$

Where, $i = 0.10$

$n = 5$

4. Column 4: **Present value of \$1 (reversion)**—The present value of one dollar (*reversion*) is the amount that must be invested now in order to receive \$1 in the future (discounting). A **reversion** is a return of rights that are due some time in the future. For example, in order to accumulate \$1 at the end of 5 years, \$0.62 must be invested at a 10% annual interest rate compounded annually. Calculate this by multiplying \$1 by the present value of \$1 factor of 0.620921. (See column 4 of the table in Figure A.7.)

$$\text{Monthly payment to amortize \$1: } \frac{0.005833}{1 - \frac{1}{(1 + 0.005833)^{360}}}$$

$$\frac{0.005833}{1 - (0.123206)} = \frac{0.005833}{0.876794} = 0.006653$$

Where, $i = 0.07 \div 12$

$n = 30 \times 12$

It is very easy to calculate the monthly mortgage payment on a 30-year loan borrowed at an annual rate of 7%. Simply multiply the loan amount (present value) by the payment to amortize $1 factor.

For a $250,000 loan amount, the monthly mortgage payment for 30 years will be $1,663.25 ($250,000 × 0.006653). After the last month in the 30th year, the amount due (future value) will be 0. At that time, the loan will be fully amortized.

Financial Calculators

A table for each and every possible combination of compounding intervals (holding periods) and interest rates is needed in order to calculate the six functions of a dollar. Obviously, one would have to have access to a very large collection of tables. Rather than using tables and long mathematical calculations, financial calculators and computer programs can perform all six functions of a dollar calculations. It is important to remember that a solution that is calculated with a "given" factor, or one that was obtained from an Ellwood Table, will differ slightly from a solution calculated using the financial function keys on a financial calculator.

There are a variety of financial calculators available on the market today. Regardless of the brand of financial calculator, the following keys would be used:

n—time period

i—interest rate*

FV—future value

PV—present value

PMT—payment (paid out or received)

*Note: When entering interest rate into most financial calculators, the number is entered as is; the calculator knows that it is a percentage and therefore converts it within. For example, 10% is entered as 10, not 0.10.

In Practice

What is the monthly payment required to fully amortize a $350,000 loan over 15 years, assuming a 7% annual interest rate? The compound interest factor for 7% interest compounded monthly for 15 years is 0.008988.

Solution:

Monthly payment = $3,145.80; (0.008988 × $350,000)

Solution (Calculator):

n = 180 (15 × 12)

i = 0.5833 (7 ÷ 12)

FV = 0 (implied)

PV = $350,000

Solve for PMT

Payment = $3,145.90*

* Note: The difference in the solutions is due to rounding.

In Practice

You need to borrow $40,000. You qualify for a $40,000 loan to be paid in full at the end of 10 years. The annual interest rate is 7.75% compounded monthly. The compound interest factor for 7.75% interest compounded monthly for 10 years is 0.012001. Calculate what you would need to pay each month for 10 years in order to pay off this loan.

Solution:

Just multiply the amount borrowed (present value) by the payment to amortize $1 factor given.

$40,000 × 0.012001 = $480.04

Therefore, after the last month of the 10th year, the amount due (future value) will be 0; at that time, the loan will be fully amortized.

Solution (Calculator):

$n = 120\ (10 \times 12)$

$i = 0.645833\ (7.75 \div 12)$

FV = 0 (implied)

PV = $40,000

Solve for PMT

Payment = $480.04

■ In Practice

What will a $5,000 investment at a 5% annual interest rate compounded annually grow to after 6 years? The future value of $1 factor for 5% annual compounding for 6 years is 1.340096.

Solution:

Multiply $5,000 (present value) by the future value of $1 factor given.

$5,000 × 1.340096 = $6,700.48

Therefore, $5,000 will grow to $6,700.48 after 6 years of compounding at an annual interest rate of 5%.

Solution (Calculator):

$n = 6$

$i = 5$

PV = $5,000

PMT = 0 (implied)

Solve for FV

Future value = $6,700.47*

*Note: The difference in the solutions is due to rounding.

This discussion, while brief, introduced the financial concepts used in the income approach. Courses and texts dedicated entirely to this topic are available to appraisers, as proficiency in advanced financial calculations is necessary for certain appraisal assignments.

Within each approach to value, the various mathematical calculations discussed in this section apply. For example, the *cost approach* utilizes area calculations for land as well as area, volume, and perimeter calculations for improvements.

Necessary calculations within the *sales comparison approach* include adjustments using a paired sales analysis (a methodized process to extract adjustments) and reconciling to a value within this approach.

The *income approach* employs income capitalization, gross rent multiplier calculations, and six functions of a dollar.

A proficiency of all of these calculations within this section will be extremely helpful when solving case problems within each approach discussed in this text.

LESSON 4 REVIEW QUESTIONS

1. Time _____ of _____ states that money available today is worth more than the same amount in the future.

2. Performing compounding and discounting calculations involves the variable of
   a. future value.
   b. time period.
   c. interest rate.
   d. All of the above

3. _____ is the process of converting an estimated future income into a present value.

4. The principal amount plus any accrued interest is the
   a. future value.
   b. present value.
   c. compound interest.
   d. simple interest.

5. _____ is interest that is computed on the original principal plus any the accrued interest.

6. What is the monthly payment required to fully amortize a $200,000 loan over 30 years, assuming a 7.25% annual interest rate? The compound interest factor for 7.25% interest compounded monthly for 30 years is 0.006822.

7. The present value of $1 factor is also referred to as the _____ _____ factor.

LESSON 4 REVIEW ANSWERS

1. value; money

2. d; all of the above

3. discounting

4. future value

5. compound interest

6. monthly payment = $1,364.40 (0.006822 × $200,000)

   Calculator Solution:

   n = 360; (30 × 12)

   i = 0.604167 (7.25 ÷ 12)

   FV = 0 (implied)

   PV = $200,000

   Solve for PMT

   Payment = $1,364.35

   *Note: The difference in the solutions is due to rounding.

7. reversion factor

APPENDIX REVIEW QUESTIONS

1. $13 - (-4) =$
2. $-3 + 4 - 6 + 9 =$
3. $2\,(8 - 3) - (-3) + 7^2 - 20 \div 2 =$
4. $3 \times \{24 - [3 + (4 \times 2)]\} =$
5. $-3 - 16 - 13 =$
6. Round 0.1497 the nearest 1,000th
7. Round 5.249 to the nearest 10th
8. $(0.12)^2 =$
9. $4^1 \times 4^2 =$
10. The reciprocal of $5/6 = ?$
11. $(1/3) \div (3/4) = ?$
12. 30 cubic yards = _____ cubic feet
13. A vacant commercial lot sold for $2,400 per front foot or $288,000. What was the frontage of the lot?
14. The mean, median, and mode are common measures of
    a. variance.
    b. central tendency.
    c. variability.
    d. deviations.
15. Determine the range of the following data set:

    {66, 55, 77, 66, 52, 80}

16. A statistic is a characteristic of a _____.

17. A _____ is oftentimes too large to measure.

18. _____ is interest earned on the principal only, not including any accrued (accumulated) interest.

19. The present value of $1 per period and the future value of $1 per period factors are also referred to as _____ factors.

20. _____ is the process of converting today's money into a future value.

SECTION REVIEW ANSWER KEY

SECTION 1 REVIEW ANSWERS

1. **a.** Real estate is the physical land and improvements.

2. **a.** A site is land that has improvements that would support a specific use.

3. **b.** Topography is the surface contour of the land.

4. **b.** Plottage is the incremental value increase as a result of combining two or more parcels. Assemblage is the act or process.

5. **a.** Assemblage is the act or process of combining two or more parcels under a single ownership to create a larger tract with greater utility. Plottage is the incremental value increase as a result of combining two or more parcels.

6. **b.** Land on which no improvements have been made is known as raw land or unimproved land. A site has some improvements that would support a specific use. A subdivision is a tract of land that has been improved with streets, utilities, and so on, and is further divided into lots.

7. **d.** A neighborhood is defined as a geographic area delineated by geographic or political boundaries that is characterized by having complementary land uses.

8. **b.** Plottage is the incremental value increase that is commensurate with the additional utility created by combining two or more parcels under a single ownership. The act or process of combining two or more parcels under a single ownership to create a larger tract with greater utility is called assemblage.

9. **c.** The protected classes are considered to be irrelevant as neighborhood descriptions.

SECTION 2 REVIEW ANSWERS

1. **b.** Both the ground rent capitalization method and the land residual method use the IRV formula.

2. **d.** All of the above are true. With the ground rent capitalization method, the net income associated with the ground lease is capitalized to derive a land value. There is an active market for ground leases. The direct capitalization rate can be derived from the market and the tenant will want the lease to run as long as the estimated useful life of the building.

3. **d.** Percentage adjustments should be made in the following order: property rights conveyed, financing, conditions of sale, and market conditions (time).

4. **a.** Unit of comparison = Sales price ÷ Element of comparison

5. **b.** If the land to building ratio is 1:6, there are seven total parts (one to the land and six to the building). Therefore, the land is 1/7 of the total property value.

6. **a.** The total estimated land value is $39,833. The ratio of land to building is 1:5 for a total of six parts. Divide the total property value by the total number of parts to determine what the value of one part is: $239,000 ÷ 6 = $39,833.

7. **c.** The land to building ratio is 1:4, making five total parts. This makes the land 1/5 of the total property value and the building 4/5 of the total property value.

8. **c.**

$1,000,000	Replacement cost new (RCN)
– 300,000	30% estimated accrued depreciation
$700,000	Depreciated cost of the improvements
$885,000	Comparable property
– 700,000	Depreciated cost of the improvements
$185,000	Value

9. **d.** All of the above are true.

10. **a.** ($300,000 × 15) – ($185,000 × 15) = $1,725,000

11. **b.** $30,000 ÷ 0.10 = $300,000

12. **d.** ($5,000 × 12) ÷ 0.08 = $750,000

13. **c.** Income (I_B) = Value (V_B) × Capitalization rate (R_B)

= ($152 × 3,400) × 0.11

= $516,800 × 0.11

= $56,848 ($I_B$)

Land income (I_L) = Total property income (I_O) – Building income (I_B)

$9,152 = $66,000 – $56,848

Value (V_L) = Income (I_L) ÷ Capitalization rate (R_L)

= $9,152 ÷ 0.09

= $101,689 ($V_L$)

14. **a.** $56,848

Income (I_B) = Value (V_B) × Capitalization rate (R_B)

= ($152 × 3,400) × 0.11

= $516,800 × 0.11

= $56,848 ($I_B$)

15. **d.** $9,152

Land Income (I_L) = Total property income (I_O) – Building Income (I_B)

$66,000 – $56,848

= $9,152

SECTION 3 REVIEW ANSWERS

1. **b.** The principle of substitution related to the cost approach states that no person is justified paying more for a property than they can build new without any undue delay.

2. **c.** Functional obsolescence is a form of depreciation. Functional depreciation, economic or external depreciation, and physical obsolescence are fictitious terms.

3. **b.** Functional obsolescence refers to the market's response to a superadequacy or a deficiency. Physical depreciation is the result of general wear and tear.

4. **d.** External obsolescence is a value loss that is realized as a result of the market's negative response to something that is outside of the property lines. Functional obsolescence is a value loss as a result of the market's negative response to some functionality of the property. Physical depreciation is the loss in value as a result of general wear and tear.

5. **d.** In the cost approach method, the replacement cost, or the reproduction cost, of a building is first determined, then accrued depreciation is subtracted, and finally the value of the land or site as though vacant is added.

6. **c.** A loss in value as a result of an overimprovement is known as a superadequacy.

7. **b.** The straight-line depreciation method is the simplest method and is calculated by dividing the cost new by the useful life (expected life) to find an equal annual percentage of depreciation over the life of the asset.

8. **c.** The straight-line depreciation method is the simplest method and is calculated by dividing the cost new by the useful life (expected life) to find an equal annual percentage of depreciation over the life of the asset ($200,000 ÷ 50 = $4,000).

9. **d.** Because total economic life equals effective age plus remaining economic life, both b and c are correct formulas.

10. **b.** Chronological age is defined as the age that the property has actually been in existence from the time of its original construction. Therefore, 36 years is the chronological, or actual, age of this home.

11. **c.** Total economic life equals effective age plus remaining economic life (TEL = EA + REL).

12. **b.** The breakdown method is the most detailed and comprehensive method of calculating depreciation. The age-life method does not consider curable and incurable items. The unit-in-place method determines replacement/reproduction costs new, not depreciation. The straight-line depreciation method is the simplest method.

13. **c.** Quantity survey method is the most comprehensive method of costing. The costs are estimated by accounting for every item into a project, including the cost of individual units of materials, labor, fees, and so on. The economic age-life method determines depreciation not cost new.

The unit-in-place method costs the major components of a structure, such as the flooring, roof, wall structure, foundation, plumbing, heating and air-conditioning, electrical, and so on. The square-foot method is similar to the unit-in-place method of costing except the property is broken down into a square-foot unit of comparison.

14. **d.** Accrued depreciation refers to the loss of value from all causes including physical depreciation, functional obsolescence, and external obsolescence.

15. **c.** Chronological age is defined as the age that the property has actually been in existence from the time of its original construction. Therefore, the chronological, or actual, age of this building is eight years.

APPENDIX REVIEW QUESTIONS

1. $13 - (-4) = 13 + (+4) = 13 + 4 = 17$

2. $-3 + 4 - 6 + 9 = 4$

3. $2\,(8 - 3) - (-3) + 7^2 - 20 \div 2 =$

   $2 \times (5) - (-3) + 7^2 - 20 \div 2$

   $2 \times [5] - (-3) + 49 - 10$

   $10 + 3 + 49 - 10 = 52$

4. $3 \times \{24 - [3 + (4 \times 2)]\}$

   $3 \times \{24 - [3 + (8)]\}$

   $3 \times \{24 - [11]\}$

   $3 \times \{13\} = 39$

5. $-3 - 16 - 13 = -(3 + 16 + 13) = -(32) = -32$

6. Round 0.1497 to the nearest 1,000th = 0.150

7. Round 5.249 to the nearest 10th = 5.2

8. $(0.12)^2 = 0.12 \times 0.12 = 0.0144$

9. $4^1 \times 4^2 = 4^{1+2} = 4^3 = 4 \times 4 \times 4 = 64$

10. The reciprocal of $5/6 = 6/5$; $5/6 \times 6/5 = 30/30 = 1$

11. $(1/3) \div (3/4) = (1/3) \times (4/3) = \frac{1 \times 4}{3 \times 3} = \frac{4}{9}$

12. 30 cubic yards = 810 cubic feet; (30 × 27) cubic feet

13. $2,400 × frontage = $288,000

$$\frac{\$2{,}400 \times \text{frontage}}{\$2{,}400} = \frac{288{,}000}{\$2{,}400}$$

frontage = 288,000 ÷ 2,400 = 120'

14. **b.** central tendency

15. Range = (80 – 52) = 28

16. sample

17. population

18. Simple interest

19. annuity

20. Compounding

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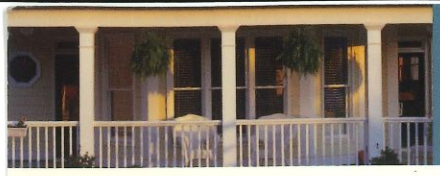

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